APPLICATIONS OF THE SECOND ORDER PERTURBATIONS THEORY AND THE GREEN FUNCTION METHOD TO THE ECONOMIC UNITS

Sever SPÂNULESCU*

Abstract. *A set of similarities between an economic unit and an atomic system are drawn and the corresponding quantitative relationships are established. As some of the most important atomic processes are successfully studied in the framework of the Quantum Electrodynamics, the present work proposes applying some of the specific mathematical instruments of this domain to the economic studies, in particular the perturbations theory and the Green function method. Based on the analytical formulae obtained for the photoionization, some conclusions concerning the probability of migration of the employees from an economic unit are claimed, mainly the peculiar phenomenon of the probability of migration decreasing when the external offer increases.*

Keywords: *quantum electrodynamics, Hamiltonian, actual value operator, perturbations, Green function, photoionization*.

1. Introduction

The complexity of the economic systems involves often some very difficult mathematics, most of them having analytical solutions only for very simple models that cannot supply realistic solutions. Some specific approximation methods for solving the equations associated with complex models have been developed, especially for the cases involving partial derivative equations and nonlinear terms [1, 2]. Taking into account that the economic systems usually display a high synergy, it is expected that a realistic model has to be a self-consistent one, in which every part is influenced by all the others, so that any calculations have to be conducted in an iterative way.

* Hyperion University of Bucharest, 169 Calea Călăraşilor, Street, 030615, Bucharest, Romania, Phone: 401-323 41 67; 401-323 23 95, Fax: 401-323 23 95, and by e-mail to: severspa2004@yahoo.com.

This situation is also met in the atomic systems, where all the parts (electrons and nuclei) are evolving in an overall field generated by each of them – the self-consistent field [3-5]. The interactions of such systems with the exterior are well studied using the methods of the quantum electrodynamics (QED) [6] and quantum mechanics. Remarkable progresses were made in the last decades for approximately solving the huge systems of differential equations that arise in such calculations and several efficient mathematical methods have been specially developed for these particular fields [7-9].

2. An atomic model of an economic unit

In a similar way that we describe the evolution of an electron bounded in an atom, we may describe the activity of one employee as a function $\psi_n(\mathbf{x})$ whose absolute square $|\psi_n(\mathbf{x})|^2$ indicates the influence (or the value) of that particular employee in various zones of the institution. The variables that compose the vector **x** are specific to the respective institution so we shall treat them as generic. For a short enough period of time the system may be considered as stationary, so that the variables vector does not include time. In this steady state form, the employees have specific values for the institution, E_n which belong to a discrete set, similar to the quantified values of the energy for atom bound electrons.

Let us consider an institution *A*, with an overall value Z_{eff}, whose function chart contains *n* levels, each of them described by an eigenvector $\psi_i(\mathbf{x})$, forming the set $\{\psi_1(\mathbf{x}),\psi_2(\mathbf{x}),...,\psi_n(\mathbf{x})\}$, where $\psi_1(\mathbf{x})$ is the most important function, and $\psi_n(\mathbf{x})$ the least important. Each level is occupied by one or several employees. The institution has a number of *N* departments, where each employee has a certain value $|\psi_n(\mathbf{x})|^2$, according to his attributions. The relationship between the functions $\psi_n(\mathbf{x})$ and the values E_n may be written as an eigenvectors and eigenvalues problem, in a similar way as the time independent Schrodinger equation:

$$H\psi_n(\mathbf{x}) = E_n\psi_n(\mathbf{x}). \tag{2.1}$$

In this equation $\psi_n(\mathbf{x})$ are the eigenvectors and E_n the eigenvalues of an operator *H*, which may be interpreted as the Hamiltonian of the system. The macroeconomic theory deals with this type of operator (mainly used in physics) for the economic systems as the actual value operator [10, 11].

Among the many powerful mathematical tools dealing with such operators, we shall focus on the perturbation theory [6], which may successfully describe the interactions of the atomic system with the exterior and the modifications that may occur, especially in the scattering theory. Some elementary processes are quantitatively well described in the framework of QED using the perturbation theory, using the self-consistent field method [3, 4] or the Green function method [6]. We consider that some of the major mathematical techniques involved by this approach in physics may be ported to the economic systems theory, and the present paper marks our first attempt in this direction.

Thus, we shall refer to the photoionization of the atoms, an inelastic process involving the perturbation of the atomic structure by an incoming photon which leads to the eviction of an electron. A similar situation may be observed in an economical unit, when an external perturbation may lead to the dismissal of an employee (either wanted or not). In fact, in QED, the perturbation may lead to the removal of an electron with a certain probability, and the noticed magnitude of the process is described by its effective cross-section, which describes the likelihood of actually producing the effect. It may be calculated from the matrix element of the process, which is obtained as an approximation by the perturbation theory [7, 8].

The standard procedure for obtaining the matrix element in QED is to develop at total Hamiltonian of the perturbed system in a power series including the unperturbed system's Hamiltonian and the first order perturbation, the second order perturbation and so on:

$$H = H^{(0)} + \lambda H^{(1)} + \lambda^2 H^{(2)} + ... \tag{2.2}$$

Here, the unperturbed system's Hamiltonian satisfy the Schrodinger equation for every unperturbed eigenvector and eigenvalue:

$$H^0 \psi_n^0 = E_n^0 \psi_n^0. \tag{2.3}$$

Then, one considers that the perturbed system's eigenvector and eigenvalue may be expressed in similar terms, containing the various order perturbations:

$$\psi_n = \psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + ... \tag{2.4}$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} \tag{2.5}$$

and the perturbed system obeys a similar eigenvectors and eigenvalues equation:

$$H\psi_n = E_n\psi_n. \tag{2.6}$$

Plugging in the expressions (2.3)-(2.5) in (2.6) we obtain:

$$(H^{(0)} + \lambda H^{(1)} + \lambda^2 H^{(2)} + \lambda^3 H^{(3)} + ...)(\psi_n^{(0)} + \lambda\psi_n^{(1)} + \lambda^2\psi_n^{(2)} + \lambda^3\psi_n^{(3)} + ...) =$$

$$= (E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \lambda^3 E_n^{(3)} + ...)(\psi_n^{(0)} + \lambda\psi_n^{(1)} + \lambda^2\psi_n^{(2)} + \lambda^3\psi_n^{(3)} + ...). \tag{2.7}$$

Now, the key of the approximation is to limit these infinite expansions to a certain order, according to the desired precision goal and thus simplifying the overall calculations. In practice, one may retain the first two terms (and obtain the results in the first order of the perturbations theory), or the first three terms (obtaining the results in the second order of the perturbations theory).

For simplicity, we shall illustrate the method for obtaining the matrix element by referring in the following only to the first order perturbation theory. The eigenvectors and eigenvalues equation (2.7) becomes:

$$(H^{(0)} + \lambda H^{(1)})(\psi_n^{(0)} + \lambda\psi_n^{(1)}) = (E_n^{(0)} + \lambda E_n^{(1)})(\psi_n^{(0)} + \lambda\psi_n^{(1)}). \tag{2.8}$$

By identifying the terms in λ in both sides of the equation we obtain:

$$H^{(0)}\psi_n^{(1)} + H^{(1)}\psi_n^{(0)} = E_n^{(0)}\psi_n^{(1)} + E_n^{(1)}\psi_n^{(0)}. \tag{2.9}$$

For further calculations we shall adopt some usual conditions for the Hamiltonian operator and the eigenvectors, considering them as parts of a Hilbert space and defining the scalar product specific to this kind of space. By using the bra-ket notation for the vectors and denoting with $|n_i\rangle = \psi_n^{(i)}(\mathbf{x})$ the function ψ_n in the order *i* of the perturbation, we may write:

$$\langle n_i \mid n_j \rangle = \int_D \psi_n^{(i)}(\mathbf{x})\psi_n^{(i)*}(\mathbf{x})d\mathbf{x} = \delta_{ij} \tag{2.10}$$

where we considered that all the vectors form an orthonormal system and $\psi_n^{(i)*}$ is the complex conjugate of the function $\psi_n^{(i)}$.

With these notations, equation (2.9) becomes:

$$(H^{(0)} - E_n^{(0)}) \mid n_1 > = (E_n^{(1)} - H^{(1)}) \mid n_0 > . \tag{2.11}$$

By left multiplying with the bra $< n_0 |$ we obtain:

$$\langle n_0 | H^{(0)} | n_1 \rangle - \langle n_0 | E_n^{(0)} | n_1 \rangle = \langle n_0 | E_n^{(1)} | n_0 \rangle - \langle n_0 | H^{(1)} | n_0 \rangle. \quad (2.12)$$

Taking into account eq. (2.3) and (2.10) we see that the left member of the equation (2.12) equals zero:

$$\langle n_0 | H^{(0)} | n_1 \rangle = \langle n_0 | E_1^{(0)} | n_1 \rangle = E_1^{(0)} \langle n_0 | n_1 \rangle = 0. \quad (2.13)$$

Thus, from the right member of eq. (2.12) we obtain the first order corrections of the eigenvalues:

$$E_n^{(1)} = \langle n_0 | H^{(1)} | n_0 \rangle. \quad (2.14)$$

For obtaining the first order corrections to the eigenfunction $| n_1 >$, we express it as a linear combination of the zero order eigenfunctions $| k_0 \rangle$:

$$| n_1 >= \sum_k c_k | k_0 >= \sum_k \langle n_1 | k_0 \rangle | k_0 \rangle. \quad (2.15)$$

By left multiplying eq. (2.11) with the ket $< k_0 |$ we may write:

$$\langle k_0 | H^0 - E_n^0 | n_1 \rangle = \langle k_0 | E_n^1 - H_\varepsilon^1 | n_0 \rangle. \quad (2.16)$$

The left member of this equation may be written as:

$$\langle k_0 | H^0 - E_n^0 | n_1 \rangle = \langle n_1 | H^0 - E_n^0 | k_0 \rangle = \langle n_1 | E_k^0 | k_0 \rangle - \langle n_1 | E_n^0 | k_0 \rangle =$$

$$= (E_k^0 - E_n^0) \langle n_1 | k_0 \rangle. \quad (2.17)$$

Taking into account that, according to eq. (2.10), $\langle k_0 | E_n^1 | n_0 \rangle = = E_n^1 \langle k_0 | n_0 \rangle = 0,$ from eq. (2.16) we get:

$$\langle n_1 | k_0 \rangle = \frac{\langle k_0 | H^1 | n_0 \rangle}{E_n^0 - E_k^0}. \quad (2.18)$$

By plugging in this expression into eq. (2.15) we finally obtain the eigenfunction's correction in the first order of the perturbations theory:

$$| n_1 >= \sum_{k \neq n} \frac{\langle k_0 | H^1 | n_0 \rangle}{E_n^0 - E_k^0} | k_0 >= \sum_{k \neq n} \frac{H_{kn}^{(1)}}{E_n^{(0)} - E_k^{(0)}} | k_0 > \quad (2.19)$$

where we denoted with $H_{kn}^{(1)} = \langle k_0 | H^1 | n_0 \rangle$ the matrix element of the transition from the *k* level to the *n* level.

Using a similar (but more extended) procedure one may find the second correction to the eigenvalue, and so on:

$$E_n^{(2)} = \left\langle n_0 \mid H^{(2)} \mid n_0 \right\rangle + \sum_{k \neq n} \frac{H_{nk}^{(1)} H_{kn}^{(1)}}{E_n^{(0)} - E_k^{(0)}}. \tag{2.20}$$

In QED, the main difficulty for estimating the corrections to the energies and wave functions is to calculate the sum over the complete set intermediate energies that appears in the expression (2.20).

A very powerful method for overcoming the difficulties associated with these calculations is to replace this sum with the Green function [13], defined by the equation:

$$(H^{(0)} - E_k^{(0)}) G_0(\vec{r}_2, \vec{r}_1; \Omega) = \delta(\vec{r}_2 - \vec{r}_1) \tag{2.21}$$

so that:

$$G_0(\vec{r}_2, \vec{r}_1; \Omega) = \sum_{k \neq n} \frac{\left\langle k_0(\vec{r}_2) \mid n_0(\vec{r}_1) \right\rangle}{E_n^{(0)} - \Omega}. \tag{2.23}$$

The Green function is specific to the system's Hamiltonian and generally it also may be difficult to calculate. However, it is already known for some particular situations, as the central field systems, characterized by a Coulombian attractive force, and could be used in similar economic systems.

3. The Coulombian form of the bond

It is obvious that the long term stability of an economic unit is provided by a certain "attraction force" that the unit operates on each employee. If one would define a "distance" r_n of an employee *n* from the core of the economic unit, it would be reasonable to consider that the bonding energy derived from this attraction force is inverse proportional with this distance, as in the Coulombian case in physics. The most central employees feel the strongest attraction force and have the higher absolute value potential energy, while the most peripheral employees feel the weakest one and have the smallest absolute value potential energy. Thus we may write the potential energy derived from the attraction force that the economic unit exercises on the employee *n* as:

$$E_n = -K \frac{Z_{eff}}{r_n} z_n \tag{3.1}$$

where Z_{eff} is the overall value of the enterprise and z_n is the value of the employee *n*. The sign is used to suggest that the force is an attractive one, so that the employees find themselves in a potential well, with negative quantized energy levels, just as the electrons in an atom.

Considering the normal case that the remuneration of an employee is equal to its value for the economic unit, we may also write:

$$E_n = -K \frac{Z_{eff}}{r_n} |\psi_n(\mathbf{x})|^2 . \tag{3.2}$$

This is exactly the Coulombian form of the bonding energy of a single electron in an atom, suggesting that, for describing some economic processes, one may use the Coulombian Green function given by Schwinger [12]:

$$G_0(\vec{p}_1, \vec{p}_2; \Omega) =$$

$$= \frac{m}{2\pi^2} X^3 \left(\frac{ie^{i\pi\tau}}{2\sin\pi\tau} \right) \int_1^{(0+)} d\rho\, \rho^{-\tau} \frac{d}{d\rho} \left\{ \frac{1-\rho^2}{\rho} \frac{1}{\left[X^2(\vec{p}_1 - \vec{p}_2)^2 + (p_1^2 + X^2)(p_2^2 + X^2) \frac{(1-\rho)^2}{4\rho} \right]^2} \right\}.$$

4. Matrix elements for atomic processes in the second order of the perturbation theory and their interpretation for economic systems

Using the Green function method, in the second order of the perturbation theory, and the QED framework, the matrix elements for the 1s, 2s and 2p atomic processes has been recently calculated [13-15]. Also, *via* the optical theorem[1], analytical formulae for the total cross-sections for the photo effect and pair production with the electron bounded in the atom could be obtained, with a very good accuracy compared with other methods (as the *ab initio* calculations results presented in literature [16,17]).

[1] The optical theorem: The total cross section for the photoeffect is proportional with the imaginary part of the elastic forward scattering amplitude [6].

Such formulae involve several physical parameters that may hardly be associated with the economic theory, so that we should only present the main steps for obtaining them, and interpret the qualitative results.

Thus, the Kramers-Heisenberg-Weller matrix elements for elastic processes in the second order of the perturbations theory are:

$$M_{if}(\Omega_1) = -m \lim_{\varepsilon \to 0} \sum_n \frac{< f \mid e^{-i\vec{k}_2 \cdot \vec{r}_2} (\vec{\alpha} \cdot \vec{s}_2) \mid n >< n \mid e^{i\vec{k}_1 \cdot \vec{r}_1} (\vec{\alpha} \cdot \vec{s}_1) \mid i >}{E_n - (\omega_1 + \gamma m + i\varepsilon)} \qquad (4.1)$$

where the most important parameters are: ω_1 is the energy of the incoming perturbation (photon), $\vec{k}_j = \omega_j \vec{v}_j$ and $\vec{s}_j$ are the momentum and the polarization, $| i >$ and $| f >$ are the Dirac spinors (wavefunctions of the bond electron), *m* is its mass and *E* is its energy.

As we may see, this formula implies the summing over the complete set of intermediate states charactrized by the eigenfunctions $| n >$ for both positive and negative energies, which may be replaced by the Coulombian Green function:

$$G(\vec{r}_2 \cdot \vec{r}_1; \Omega) = \sum_n \frac{| n >< n |}{E_n - \Omega} \qquad (4.2)$$

for the Dirac equation:

$$G(\vec{r}_2 \cdot \vec{r}_1; \Omega) =$$

$$= \frac{1}{2m} \left(i\vec{\alpha} \cdot \nabla_2 - \beta m - \frac{\alpha Z}{r_2} - \Omega \right) \left[I + \frac{1}{2\Omega} \vec{\alpha} (\vec{P}_2 + \vec{P}_1) \right] G_0(\vec{r}_2 \cdot \vec{r}_1; \Omega) \qquad (4.3)$$

where $G_0(\vec{r}_2 \cdot \vec{r}_1; \Omega)$ is the Green function for the Schrödinger type equation:

$$\left(-\frac{1}{2m} \nabla_2 - \frac{\alpha Z \Omega}{m} \frac{1}{r_2} + \frac{m^2 - \Omega^2}{2m} \right) G_0(\vec{r}_2 \cdot \vec{r}_1; \Omega) = -\delta(\vec{r}_2 - \vec{r}_1). \qquad (4.4)$$

Using this procedure, the matrix element may be expressed in terms of the Dirac spinors $u_\mu(\vec{p} - \vec{k})$. Considering the finite speed of the perturbations (*i.e.* the retardation), it is more convenient to use the momentum representation of the equations, after a Fourier transform. The analytical expression of the Dirac spinor is specific to a certain Hamiltonian and eigenfunction. For example, for an electron in the ground state (*K* shell), in

the Sommerfeld-Maue approximation (spin corrections neglecting), the Dirac spinor was given by Boyer [18]:

$$u_{\mu}(\vec{p}-\vec{k}) = \left[a(\vec{p}-\vec{k}) + \frac{\vec{\alpha}(\vec{p}-\vec{k})}{2m} b(\vec{p}-\vec{k}) \right] \chi_{\mu} : \ \chi^{+}_{\frac{1}{2}} = (1000),$$

$$\chi^{+}_{-\frac{1}{2}} = (0100). \tag{4.5}$$

With these elements, the transition matrix element for elastic scattering may be expressed in a closed form by neglecting the higher order terms and using only those corresponding to the second order perturbation theory.

For the *K* shell electrons, the analytical form of the elastic scattering amplitudes have been recently calculated [14]. They imply some higher transcendental functions of Appell type $F_1(a-\tau; a, a; a+1-\tau; x_1, x_2)$, which are no more computational challenging and offer a very good agreement (within some percent) with other experimental and theoretical results [16,17]:

$$M^{NR}_{GC} = M_{GC}(\omega,\theta)(\vec{s}_1\vec{s}_2) + N_{GC}(\omega,\theta)(\vec{s}_1\vec{v}_2)(\vec{s}_2\vec{v}_1) \tag{4.6}$$

$$M_{GC}(\omega,\theta) = [\,\vartheta - P_{GC}(\Omega_1,\theta) - P_{GC}(\Omega_2,\theta)] \tag{4.7}$$

$$N_{GC}(\omega,\theta) = -[Q_{GC}(\Omega_1,\theta) + Q_{GC}(\Omega_2,\theta)] \tag{4.8}$$

$$P_{nr}(\Omega_1) = 2^3(\alpha Z)^8 \frac{m^4}{\omega^4}\left(\frac{\omega}{\omega_{th}} - 1\right)^{\frac{3}{2}} \frac{F_1(2-\tau_1^{nr}; 2, 2; 3-\tau_1^{nr}; x_1^{nr}; x_2^{nr})}{2-\tau_1^{nr}} ie^{4ix_0^{nr}} \tag{4.9}$$

$$Q_{nr}(\Omega_1) =$$

$$= -2^5(\alpha Z)^{10} \frac{m^4}{\omega^4}\left(\frac{\omega}{\omega_{th}} - 1\right)^{\frac{5}{2}} \frac{F_1(3-\tau_1^{nr}; 3, 3; 4-\tau_1^{nr}; x_1^{nr}, x_2^{nr})}{3-\tau_1^{nr}} ie^{6ix_0^{nr}} \tag{4.10}$$

$$\vartheta = \left[1 + \frac{1}{\alpha^2 Z^2}\left(\frac{\omega}{m}\right)^2 \sin^2\frac{\theta}{2}\right]^{-2}. \tag{4.11}$$

In the case of the 2*s* and 2*p* subshells, the analytical expressions were also obtained [15], but they are more complicated and shall not be exposed here.

The most important result for our study is the total photo effect cross section, which describes the probability of removing and electron from the atom due to the external perturbation. Thus, by using the optical theorem:

$$\sigma^{PH} = \frac{4\pi}{\alpha}\frac{m}{\omega} r_0^2 \mid \mathrm{Im}\, M(\omega, \theta = 0) \mid = \frac{4\pi}{\alpha}\frac{m}{\omega} r_0^2 \mid \mathrm{Im}\, P_{nr}(\Omega_1, \theta = 0) \mid \quad (4.12)$$

the total photoeffect cross section for the 1s shell is:

$$\sigma_{1s}^{PH} = \frac{32}{3}\pi^2 r_0^2 E_0 m^2 \alpha^5 Z^6 \frac{E_0 + \omega}{\omega^4} \frac{e^{-|\tau_1^{nr}|[\pi = \chi_{nr}(\omega)]}}{1 - e^{-2\pi|\tau_1^{nr}|}} \quad (4.13)$$

while for the 2s subshell the total photoeffect cross section it is:

$$\sigma_{2s}^{PH} = \frac{2}{3} r_0^2 \pi^2 \alpha^5 Z^6 \frac{m^2}{\omega^2} \frac{(\mid X_1 \mid^2 + \alpha^2 Z^2 \Omega_1^2)}{\omega^2} \frac{\Omega_1}{\omega} \frac{e^{-|\tau_1[[\pi + \chi_1(\omega)]}}{1 - e^{-2\pi|\tau_1|}} \times$$

$$\times \left[1 - \frac{\alpha^4 Z^4}{20} \frac{\mid X \mid^2}{\omega^2} - \frac{4}{5}\alpha^2 Z^2 \frac{\omega_{th}^2}{\omega^2} \right]. \quad (4.14)$$

As an example, the summed photoeffect cross-sections given by these formulae for the lead 1s and 2s electrons as a function of the incoming photon energy are illustrated in figure 1.

5. Conclusions

Some conclusions may be drawn from these formulae, and they already have some correspondence with the economic domain.

Thus, by inspecting the scattering amplitudes expressions, one may prove that they do have an imaginary part only if the incoming perturbation's energy is higher than a certain threshold, which is the electron's binding energy. As the photoeffect cross section is proportional with the imaginary part of the amplitude, (according to eq. (4.12)), it follows that no eviction may appear for a perturbation below this threshold.

This is a predictable result, meaning that the removal of an employee by an external perturbation may only occur if the perturbation is higher than the affinity of that person to the current institution. So, in figure 1 one may see that no effect appears for a certain level of the electron if the incoming photon energy is below an energy threshold. For the most important cases of inner (1s and 2s) electrons, the incoming photon energy must exceed approx. 100 keV and respectively 24 keV. Thus, no inner electron is removed if the perturbation's energy is lower than the 2s

threshold, where the channel for 2s electrons' eviction opens. By further increasing the perturbation's energy, a new channel is opened at the 1s threshold and these mostly bond electrons may also be evicted. For atomic systems with more electrons, the energy thresholds are as many as the binding energies. Of course, the peripheral electrons are the most likely to be removed, even at lower perturbations, while the inner ones need a much higher perturbation.

On the other hand, an intriguing phenomenon may be noticed according to the formulae (4.13)-(4.14) and figure 1. Above the threshold, the total cross section for the photoeffect strongly *decreases* (almost exponentially) with the incoming photon energy. So, how could it be possible for an economic unit that a higher external action to be less likely to remove a certain electron, than a lower one? After all, in economic terms, this action may be measured in the remuneration and position that another economic unit offers to a certain employee of the current one. It is normal that this offer must exceed the actual level, so that the existence of the thresholds is obvious. But the perturbations theory predicts that if the offer increases, the likelihood that the removal of that particular employee to appear decreases.

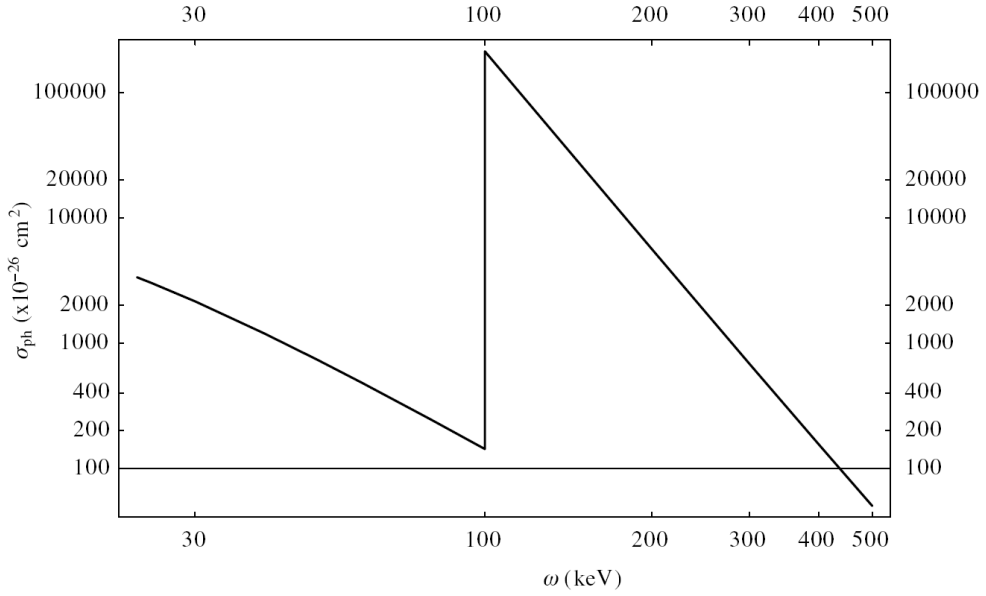

Figure 1. The summed photoeffect cross-sections for the lead 1s and 2s electrons as a function of the incoming photon energy.

In our opinion, this phenomenon must be interpreted in a statistical way. In QED, when the removal of the electron is achieved, the incoming

perturbation (photon) ceases (is absorbed). If it does not occur, the perturbation is applied to another one, maybe on another level or even another atom, and so on, till the absorption does appear. In economic terms, it may be considered that a higher offer is aimed to a higher level, so that *an employee on a lower level in an economic unit is not very often expected to correspond to a higher offer*. The external action is intended to seek the removal of an employee from an as much as possible high level in a statistical collective of economic units, and is rarely "absorbed" by a lower level employee. In other words, a certain external offer is more probably made for the right person, and the probability of making an offer higher than the real level of a person decreases exponentially above the threshold, just as the perturbations theory in QED predicts.

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REFERENCES

[1] F. Black, M. Scholes, *Journal of Political Economy*, **81** (3) 637-654 (1973);
[2] J. C. Hull, *Options, Futures, and Other Derivatives*, Prentice Hall, ISBN 0-13-601589-1 (1997).
[3] V. Fock, *Z. Physik* **61**, 1260 (1930).
[4] J. C. Slater, *Phys. Rev*. **81**, 385 (1951).
[5] A. Sommerfeld, *Atombau und Spektrallinien, Wellenmechanischer Ergänzungband* (Vieweg, Braunschweig, 1929) **vol. 2**, cap. V.
[6] Akhiezer A. I., Berestetskii V. B., *Quantum Electrodynamics*, Interscience, New York, 1965.
[7] Kramers H. A., Heisenberg W., *Z. Physik* **31** (1925) 681.
[8] Waller I., *Z. Physik* **51** (1928) 213.
[9] Kohn W., Sham L. J., *Phys. Rev.* **140** (1965) A1133.
[10] D. Cass, K. Shell, *Journal of Economic Theory*, **12** (1), 1976.
[11] Y. Hossein Farzin, *Review of Development Economics*, 10(3), 518-534, 2006.
[12] J. Schwinger, *Journal of Mathematical Physics* 5, 1606 (1964).
[13] A. Costescu , S. Spanulescu, *Phys. Rev.* A **73** , 022702 (2006).
[14] A. Costescu, S. Spanulescu and C. Stoica , *J. Phys. B : At., Mol.* , *Opt. Phys*. **40**, 2995 (2007).
[15] A. Costescu, K. Karim, M. Moldovan, S. Spanulescu and C. Stoica , *J. Phys. B: At. Mol. Opt. Phys.* **44** 045204 (2011)
[16] P. M. Bergstrom, T. Suric, K. Pisk, R. H. Pratt, *Phys Rev* A 48, 1134 (1993).
[17] J. H. Scofield, *Lawrence Radiation Laboratory Report No CRL 51326* (Livermore CA, 1973, unpublished).
[18] Boyer R. H., *Phys. Rev.* **117** (1960) 475.