NPV BASED DECISION CRITERIA FOR BORROWING PROJECTS AND PRINCIPLE OF CONSERVATION OF VALUE

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Abstract. *Financial viability of a project is a sine-qua-non for its acceptance for implementation. Capital budgeting techniques are methods employed for evaluation of the financial viability of a project. Net present value (NPV) is a capital budgeting technique used widely by both academicians and practitioners.*

Some academicians appear to consider that a uniform NPV based decision criterion viz. NPV > 0 can be used for evaluation of the financial viability of both investment projects and borrowing projects. However, our analysis clearly reveals that a uniform decision criterion NPV > 0 for both investment projects and borrowing projects is not a self-consistent requirement at all.

As our analysis in this article would establish, the self-consistent NPV based decision criteria for borrowing projects can be formulated as under:

NPV of a single/independent borrowing project ≤ 0.

For mutually exclusive borrowing projects, among the projects with negative NPVs, select the one with the highest NPV i.e. the one with the lowest magnitude of NPV.

We also propound a fundamental principle of conservation of value during financial transactions in a frictionless world.

Keywords: *Net Present Value (NPV), Investment and Borrowing Projects; Parity for NPV and IRR; Self-Consistent NPV-based Decision Criteria; Principle of Conservation of Value*

1. Introduction

Project appraisal and finance occupies a central position in the field of corporate finance. Technical feasibility and financial viability are the two essential requirements for acceptance of any investment project for implementation. Financial viability of a project can be evaluated by employing one or more capital budgeting techniques. Capital budgeting

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techniques are therefore of great significance in the domain of corporate finance.

IRR and NPV happen to be the two most commonly used capital budgeting techniques. However, the NPV based decision criteria for borrowing projects do not appear to have been formulated in a self-consistent manner. A separate set of criteria has been formulated in this article in order to incorporate the much needed self-consistency in the overall analytical process.

If better decisions from business are what we finally aim at, a proper starting point would be formulation of a set of logically consistent decision criteria for that purpose.

Finally, we also propound a fundamental principle of conservation of value during financial transactions in a world free from frictions and with a uniform cost of capital.

2. Net Present Value (NPV)

Net present value (NPV) for an investment project compares the aggregate present value of all future cash flows from the project under consideration with the initial investment amount associated with the project. NPV of such a project is defined as the aggregate present value of all future cash flows from the project less the amount of initial investment for the project. It is positive for an investment project if the aggregate of all present values exceeds the amount of initial investment. An investment project is said to be financially viable if its NPV works out to be positive or at-least non-negative i.e. if NPV ≥ 0. Negative value of NPV for such a project, on the other hand, means that the investment project is not viable and not worthy of being accepted for implementation.

In the context of NPV, it is important to note that NPV i.e. net value linked with a project is estimated by discounting the project related cash flows (DCF approach). Thus, while cash flows are simply additive in nature, NPVs are additive only if the discount rate remains uniform.

3. Investment and Borrowing Projects

An investment project is characterized by an initial outflow i.e. outlay of funds at $t = 0$, when counting of time for the project commences. This initial investment in a project is followed by a stream of project related future cash flows that, in order to make the project viable, are to be dominated by inflows. This would ensure a positive rate of return for the initial investment.

A borrowing project, on the other hand, is characterized by an initial inflow i.e. at $t = 0$. In the wake of this initial inflow comes a stream of project related future cash flows that, to make the project viable, are to be dominated by outflows. This would ensure a positive cost of borrowing/ or of debt capital for the borrowing company, which in turn would guarantee a positive rate of return for the counter party lender on its initial outlay of funds. After all, it is too much to expect that an act of stand-alone borrowing is to result in creation/addition of value for a borrower.

4. NPV based Decision Criteria for Selection of Projects

Prior to the modification proposed in this article, the NPV based decision criteria (Lo, 2008) for both investment and borrowing projects are:

(a) If NPV of a single/independent project is positive, only then accept the project. Otherwise, do not accept it.

(b) For mutually exclusive projects, among the projects with positive NPVs, select the one with the highest NPV.

However, for borrowing projects, these criteria face some fundamental problems. As our analysis in this paper would clearly establish, it is essential that we modify the above decision criteria for borrowing projects in order to ensure the basic self-consistency of the overall scheme of analysis.

5. Parity of NPV and IRR

If we start with any project and reverse the signs of all the project related cash flows (this will transform an investment project into its corresponding borrowing project and vice versa, without any change of the rate of discount), then NPV of the project too changes sign without any change of the magnitude of NPV. In other words, we can state that NPV has an odd parity with reference to such a transformation.

However, unlike NPV, IRR remains unchanged in the face of such a transformation. In other words, IRR can be said to have an even parity because its numerical value does not change when the signs of all the cash flows attached to a given project are reversed. However, it is worthwhile to remember at this point that, although its numerical value remains unchanged, the physical interpretation of IRR undergoes a sea change during such a transformation of signs of all project related cash flows. IRR for an investment project represents the rate of return on the initial fund outlay for such investment, whereas IRR for a borrowing project represents cost of the fund raised via such borrowing.

NPV and IRR thus have opposite parities. Accordingly, it is possible that while some problems may surface for IRR, these may not surface for NPV and vice-versa.

In order to explore the NPV related position further, let us consider the following 2 simple projects –project 1 and project 2 (Lo, 2008) as well as their combination:

	CF at t = 0	CF at t = 1 year	Project Type
Project 1	– 100 units	120 units	Investment
Project 2	100 units	– 120 units	Borrowing
Project (1 + 2)	0	0	1 investment + 1 borrowing

The required rate of return/discount rate is 10% p.a.

From the foregoing, we can observe that:

(a) Project 1 is an investment project, while project 2 is a borrowing project.

(b) If we reverse the signs of all the cash flows for project 1without any change of the discount rate, we get project 2 and vice versa.

(c) Project 1 has positive NPV of approximately 9.1 units while project 2 has negative NPV of exactly the same magnitude. The NPVs of project 1 and project 2 are thus equal and opposite. Such a perfect correlation between NPV of project 1 and NPV of project 2 is no act of coincidence. The same is guaranteed because of the odd parity of NPV towards reversal of signs of all project-related cash flows without any change of the cost of capital.

(d) Cash flows for the two projects considered above are rather simple in nature. However, our findings based on these can be easily extended for any generalized sequence of cash flows, and are thus valid for analysis of all kinds of projects. In other words, under all circumstances, an investment project and its corresponding borrowing project (obtained by simply changing the signs of all project related cash flows, keeping at the same time the cost of capital unchanged) will have equal and opposite NPVs. Therefore, to stipulate NPV > 0 as a decision criterion for both an investment project and the corresponding borrowing project is an absurd requirement.

That an investment project and its corresponding borrowing project must have always equal and opposite NPVs can also be verified independently from the following consideration:

If we consider the combined project (project 1 + project 2), all the cash flows for this project are zero. Hence, the NPV for the combined project cannot but be zero. In other words, NPV (project 1 + project 2) = 0.

Further, for any given company i.e. for a uniform discount rate, NPV for its various projects is known to be additive in nature i.e. NPV (project 1 + project 2) = NPV (project 1) + NPV (project 2). Therefore, NPV (project 1) + NPV (project 2) = 0. In other words, NPV (project 1) and NPV (project 2) must have equal and opposite values.

This condition is satisfied not because of any chance or fortuitous coincidence arising from a special kind of cash flow pattern. It needs to be satisfied universally, irrespective of the cash flow pattern for the projects. Thus NPV (project 1) and NPV (project 2) must be equal and opposite under all circumstances. So, it is mathematically impossible to satisfy the condition NPV > 0 for both an investment project and its corresponding borrowing project.

6. Fundamental Problem confronting NPV based Decision Criteria

It has been amply demonstrated in the previous paragraph that NPV has an odd parity. It means that a change in the signs of all project related cash flows without changing their magnitudes as well as the cost of capital (thereby transforming an investment project into a corresponding borrowing project and vice versa without any change of the cost of capital) leads to a change in the sign of the project NPV while leaving its magnitude unchanged. In other words, if an investment project has a NPV > 0, the corresponding borrowing project must have a NPV < 0 and vice versa.

Let us now consider two banks A and B belonging to the same risk category. Suppose bank A lends fund of 100 units, in the call money market or the repo market, to bank B. Also, suppose that there is no associated third party payment. Assuming the common opportunity cost for both A and B to be 10%, we find as under:

	Investment by Bank A r = 10%	Borrowing by Bank B
At t = 0	(-) 100 units	(+) 100 units
i. At t = 1 yr	(+) 100 units [NPV = (-) 9.1]	(-) 100 units [NPV = (+) 9.1]
ii. At t = 1 yr	(+) 105 units [NPV = (-) 4.5]	(-) 105 units [NPV = (+) 4.5]
iii. At t = 1yr	(+) 110 units [NPV = 0]	(-) 110 units [NPV = 0]
iv. At t = 1 yr	(+) 111 units [NPV = (+) 0.9]	(-) 111 units [NPV = (-) 0.9]
v. At t = 1 yr	(+) 120 units [NPV = (+) 9.1]	(-) 120 units [NPV = (-) 9.1]
vi. At t = 1 yr	(+) 125 units [NPV = (+) 13.6]	(-) 125 units [NPV = (-) 13.6]

It is clear from the above table that investor/lender A and borrower B, having identical opportunity cost/discounting factor, have always equal and opposite NPVs, and sum total of their NPVs is zero. Therefore, under all kinds of circumstances, it is impossible for both A and B to fulfil a uniform requirement like NPV > 0.

Physically also, it is quite logical that both the investor/lender and the borrower cannot gain value on account of a given set of cash flows exchanged between them. A gain for the lender can come only at the cost of the borrower. Had it been otherwise, both the lender and the borrower could have increased their values at will and up to an unlimited extent simplyby exchanging a set of cash flows between them in a manner most suited for this purpose. This is clearly impossible and is thus to be ignored. Incidentally, it may be interesting to recall at this stage that the counterparties of any swap deal have also equal and opposite values from the swap of a set of cash flows between them.

Thus, NPV has to be: either (i) zero for both the investor (A) and the borrower (B) or (ii) positive for one counterparty and negative with the same magnitude for the other counterparty.

7. Formulation of Self-Consistent NPV based Decision Criteria for Borrowing Projects

It is quite clear from the previous paragraph that, for a uniform rate of discount, both A (lender) and B (borrower) cannot have NPV > 0 at the same time during exchange of a given set of cash flows between them.

Every investment project has a corresponding borrowing project with a NPV equal and opposite to that of the investment project. Similarly, every borrowing project has a corresponding investment project having NPV equal and opposite to that of the borrowing project. Thus, in order to be realised, if all types of investment projects are to satisfy the condition NPV $\geq$ 0, the corresponding borrowing projects cannot fulfil the same criterion for NPV viz. NPV $\geq$ 0. As a matter of fact, borrowing projects should have NPV $\leq$ 0.

As we have seen above, for both the investor/lender and the borrower with identical discount rate/cost of capital, either NPV = 0 for both the counter-parties, or NPVs of the two counter parties are equal in magnitude but opposite in sign.

Out of these two possibilities, the scenario with NPV = 0 for both counter parties represents a borderline situation, which means that (i) the lender recovers from its outlay of funds (i.e. IRR for investment project of A) exactly its cost of fund (10% in the previous example) and (ii) the borrower pays on its corresponding borrowing (i.e. IRR for borrowing project of B) a rate of interest equalling its existing cost of funds (10% in the previous example). However, such a position, while being quite acceptable to the borrower, is not so for the investor/lender (i.e. A in the above illustration), because in such a situation A has to lend fund at its cost of capital without any net addition to its value (EVA and NPV = 0) on account of its investment activity under consideration.

In order to achieve the basic goal of wealth/value maximisation, investors will naturally look for projects with NPV > 0. If A has NPV > 0, B must have NPV < 0 so that their combined NPV = 0. To reiterate, it is quite impossible for both A (the investor/lender) and B (the borrower) with identical cost of capital to have NPV > 0 at the same time.

This position is also corroborated by the IRR based decision criterion for viability of an investment project viz. IRR of an investment project $\geq$ weighted average cost of capital related to funding this project. This criterion explicitly refers to both the income from investment (IRR of the investment project) and the weighted average cost of capital for the funds used for the purpose of such investment (IRR for the borrowing project if the investment were to be financed entirely by such borrowing). Since, in general, an investment proposal can be funded by both borrowing and equity, the IRR for the borrowing project gets replaced, in the most general scenario, by the weighted average cost for all the components of capital financing the project. Depending on whether the IRR for the investment project exceeds or equals the weighted average cost of capital, NPV on account of the combined investment and funding/borrowing activities is positive or zero (NPV $\geq$ 0 i.e. there is either net value accretion or at least no depletion of existing value).

Investments are always expected to result in addition of value to a company. As a matter of fact, a company's value is often considered as an aggregate of the values from its different investment projects. In such a situation, if all investment projects were to satisfy the condition NPV < 0, any company's aggregate value would have always been negative. This simply militates against the realities on the ground and is thus ruled out. Thus, investment projects must satisfy the criterion NPV ≥ 0.

Further, on a stand-alone basis, a borrowing project is not expected to add any value to the borrower company. Had it been possible to generate value through stand-alone borrowing projects, a company could become enormously valuable just by augmenting its debt burden. This clearly contradicts the ground realities and is thus not acceptable. Wealth/value maximisation may be the ultimate goal of all corporate activities. But, accumulating debts at will, cannot be a contributory channel for value maximisation. Wealth has to be created by judiciously selecting investment and borrowing avenues that would generate positive EVAs (leading to positive NPVs) for the company.

The condition for addition of net value to a company by an investment project is: IRR of the project >weighted average cost of capital for the project. The net value addition is directly related to (IRR of the investment project less weighted average cost of capital for the financing project), and the signs of IRR for investment project and cost of capital clearly indicate the directions in which they contribute to the net value of the company implementing the investment project. An investment project generates value @ IRR of the investment project, and a part of it is used up for meeting the cost of capital or funding cost, which includes the IRR of the company's borrowing project. The balance/net amount is then available for boosting the value of the company. The net value addition on account of an investment project is thus linked to (IRR of the investment project less Cost of Capital/IRR of the borrowing project).

IRR has even parity, and both IRR for an investment project and IRR for a borrowing project are positive. However, their contributions to the value (NPV) of the company come with opposite signs – viz. positive for investment project and negative for borrowing project.

All the above considerations suggest that the NPV based criterion for an investment project should be NPV ≥ 0 whereas that for a borrowing project should be NPV ≤ 0.

A borrowing project with a negative NPV of large magnitude would imply a high cost of borrowing and is thus not preferred to another mutually exclusive borrowing project with a negative NPV of smaller magnitude. So, smaller the magnitude of the negative NPV of a borrowing project, the better. This is quite a logical proposition because borrowing involves a cost that suppresses value. Thus, lower the impact of such a factor, better it is for the net value of the firm/business.

Therefore, while an investment project (NPV $\geq$ 0) with a higher magnitude of NPV is preferred to another with a lower magnitude, for borrowing projects (NPV $\leq$ 0) a smaller magnitude of NPV is preferred to a larger magnitude. But, in the domain of negative NPVs, a project with a lower magnitude of NPV is the project with a higher NPV. In other words, while comparing mutually exclusive borrowing projects with negative NPVs also, higher the NPV, the better.

To summarise the findings of our detailed analysis, we can formulate the self-consistent NPV based decision criteria for borrowing projects as under:

(a) NPV of a single/independent borrowing project $\leq$ 0 for acceptance.

(b) For mutually exclusive borrowing projects, among the projects with negative NPVs, select the one with the smallest magnitude of NPV i.e. the one with the largest NPV.

8. Mechanism for Creation of Value and Principle of Conservation of Value

(a) We have shown in paragraph 5 above that, for a uniform discount rate, an investment project and its corresponding borrowing project always have equal and opposite NPVs. We have further demonstrated in paragraph 6 above that, under all possible circumstances, the sum total of the NPVs of two counterparties is zero so long as these have identical cost of capital. Thus, NPV of A and NPV of B in the example have to be necessarily equal and opposite.

Does it mean that the combined NPV of a single company from its investment project and borrowing/financing project should always be zero? In other words, is it just not possible for a company to create value/wealth (NPV > 0) out of its investment projects and related borrowing projects? The answer to this question is quite clear. Creation/addition of net value by a company out of its investment project and related borrowing project is always possible.

In our example in paragraph 6, A and B are two different banks having similar risk profile and identical cost of capital – one lending and the other borrowing. So, the cash flows for A and B are equal and opposite.

They cannot but have identical IRR. Also, since A and B have identical cost of capital, their NPVs have to be necessarily equal and opposite.

Let us now consider two companies A and B, which have identical risk profile and identical cost of capital of 10% (like in paragraph 6). Suppose A has lent some fund to B and the IRR for such investment by A is 10%, which is also the IRR for the borrowing project of B. Thus, NPV for A on account of such lending/investment activity is zero and the NPV of B on account of such borrowing activity is also zero. The overall cost of capital of B continues to be 10% even after the fresh borrowing from A, while A fails to achieve any augmentation of its value due to the investment in B (NPV =0). This is perhaps the price paid by A for deciding to invest within the same risk category.

While A has lent funds to B within the same risk category by forgoing the opportunity to augment its value from such lending activity, there is no compulsion for B to pursue a similar course of action. The linkage of B with A ends with its borrowing project (which is also A's investment/lending project) and has nothing to do with the investment decisions of B.

As a mechanism for creation of value, suppose B decides to use the amount borrowed from A for investment in the bonds issued by C, which offer YTM of 12%.C may be having 12% as its cost of capital and is obviously rated worse than B. Then B has created value (NPV > 0) out of its investment in the bonds of C because IRR on B's investment = 12% > 10% (cost of capital of B). This practice is quite normal in the corporate world, and in corporate finance it is common knowledge that better rated companies maybuy bonds issued by worse rated companies but not the other way round.

(b) We also observe that, if all kinds of friction like payments to third parties (brokerage, syndication fee, taxes etc.) are ignored, the combined value of the counter parties having identical cost of capital remains constant during an investment/loan and borrowing activity. In other words, in a friction free world, the total value is conserved during such transactions involving counter parties with identical cost of capital.

Such a relationship between the counterparties is also known to exist between the holder of a long position and holder of a short position in the securities/derivatives market or between the counter parties of a forward contract or a swap deal in the absence of any third party payment. In such cases the gain of one counter party comes only at the cost of the other counter party. The combined value of both the counter parties remains unchanged following any movement in the market.

This is akin to the principle of conservation of energy in the physical world in the absence of frictional forces.

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