MEASURING CAPITAL MARKET EFFICIENCY WITH TOOLS OF STATISTICAL PHYSICS

Ladislav KRISTOUFEK*, Miloslav VOSVRDA**

Abstract. *We propose a new measure of capital market efficiency. The measure takes into consideration the correlation structure of the returns (long-term and short-term memory) and local dynamics of the markets (information dimension). The efficiency measure is then taken as a distance from an ideal efficient market situation. Methodology is applied to a portfolio of 40 stock indices. By distinguishing between local and global inefficiencies, we find that the total inefficiency is mainly driven by the local inefficiencies, i.e. a low information (fractal) dimension.*

Keywords: *capital market efficiency, long-range dependence, fractal dimension, entropy.*

1. Introduction

Efficient markets hypothesis (EMH) has been a central topic in financial economics for decades since the pioneering paper of Fama [12]. An efficient market is defined as the one where prices of securities fully reflect all available information. In his later article, Fama [13] divided the definition of the efficient market into three forms – weak, semi-strong and strong. Weak form states that no extraordinary profits can be made based on past prices of an asset. Semi-strong form says that market is efficient when all publicly available information are already reflected in the prices and strong form enlarges the information set to all information. Capital market is then said to be efficient of a specific form if investors cannot obtain extraordinary risk-adjusted returns given the information set. The most severe problem of EMH is that it is very hardly testable due to a joint-hypothesis problem, i.e. when the "extraordinary" returns are analyzed, we have to specify the "ordinary" returns, which means imposing

* Institute of Economic Studies, Charles University, Prague, Czech Republic, Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Prague

** *Institute of Economic Studies, Charles University, Prague, Czech Republic,* Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Prague

some market model. However, when we reject the hypothesis of capital market efficiency, we cannot be sure whether it signs a real inefficiency or it just means that the selected market model is inappropriate.

2. Methodology

Testing of EMH usually reduces to its weak form because the two stronger forms are quite hard to translate into some meaningful tests. In the econophysics literature, the EMH testing usually deals with long-range dependence [5, 6, 10, 8, 9], entropy [25], liquidity [3], non-stationarity [17], spin models [24], fractality [22], algorithmic complexity theory [14], and others.

We propose a new measure of capital market efficiency based on informational (fractal) dimension, entropy, and short and long-range dependence. Efficient market in our sense is described as a martingale (compare Samuelson's [19] and Fama's [11] approach of describing a random market), i.e. a process of logarithmic prices $X_t = \log P_t$, where P_t is an asset price, is a stochastic process with uncorrelated and zero conditional mean increments with finite variance. Expected returns of the process X_t are thus unpredictable based on the available data set. We use the fact that for a martingale process, we know the expected values of informational dimension D ($D = 1.5$), Hurst exponent H ($H = 0.5$) and the first-order autocorrelations $\rho(1)$ ($\rho(1) = 0$), and the fact that all these measures are bounded, to propose a new measure of capital market efficiency. Taking the ideal efficient market as a benchmark, we define an efficiency index EI as a distance of the specific capital market situation from the efficient market situation. By market situation, we mean a vector of estimates of D, H and $\rho(1)$. There, EI is effectively a norm of the market situation vector which measures the distance on an n-dimensional cube from its center characterized by the efficient market. Therefore, EI is defined as $EI = \sqrt{\sum_{i=1}^{n}\left(\frac{\hat{M}_i - M_i^*}{R_i}\right)^2}$, where n is a number of measures, $\hat{M}_i$ is an estimate of the i^{th} measure, M_i^* is an expected value of the i^{th} measure for the efficient market and R_i is a range of the i^{th} measure. As ranges of different measures we use vary, we standardize them so that the range is equal to one, implying a unit cube as a resulting space. For the efficient market, we have $EI = 0$, and for the least efficient market, we have

$EI = \frac{\sqrt{n}}{2}$ where n is a number of measures taken into consideration. Let us now briefly turn to the specific measures and concepts we utilize.

Long-term memory is characterized by asymptotically hyperbolically decaying autocorrelation function $\rho(k)$, i.e. $\rho(k) \propto k^{2H-2}$ for $k \rightarrow +\infty$. In economics terms, this means that shocks in a very distant past may have a significant effect on present behavior. H in the power-law decay of the autocorrelation function is the Hurst exponent, which ranges between 0 and 1. For $H = 0.5$, we have serially uncorrelated series. For $H > 0.5$, the series is persistent, and for $H < 0.5$, the series is anti-persistent. Long-term memory violates the efficient markets definition because it allows for arbitrage as shown by Mandelbrot and van Ness [16]. Connection between long-range dependence, predictability and market efficiency is documented in several econophysics studies [5, 6, 8, 9, 10] and in many cases, the less developed markets, which are expected to be less efficient, were characterized with Hurst exponents different from 0.5.

For estimation of Hurst exponent, we apply several methods as each is sensitive to different statistical properties of the series [4, 23] – detrended fluctuation analysis (DFA) [18, 15], height-height correlation analysis (HHCA/GHE) [2, 8] and detrending moving average (DMA) [1]. For DFA, we use $s_{min} = 5$, $s_{max} = T/5$ and apply both linear and quadratic filtering to obtain two estimates of the Hurst exponent. For HHCA, we use $\tau_{min} = 1$ and vary τ_{max} between 5 and 20 to obtain jackknife estimates of the Hurst exponent. Again, we use two slightly different methodologies (compare [2] and [8]) to obtain two estimates of H. For DMA, we use a centered moving average with $\lambda_{min} = 3$ and $\lambda_{max} = 21$ with a step of 2. Therefore, we obtain five estimates of H and use each of them in EI. Hurst exponent is expected to be equal to 0.5 for the efficient capital market. Deviation from this value implies possibility of extraordinary returns and arbitrage strategies [16].

The first order autocorrelation is simply estimated through a AR(1) maximum likelihood estimation. The expected value for the efficient market is equal to 0. Note that the correlation coefficient ranges from –1 (perfectly anti-correlated) to 1 (perfectly correlated) and thus has twice the range of D or H. The distance from the ideal case of 0 is thus halved when accounted for in the efficiency index.

Fractal dimension $D \in [n, n+1)$ is a measure of roughness on n-dimensional sphere; for a univariate case, $D \in [1,2)$. On a hyperplane R^{n+1}, fractal dimension is a local characteristic of the time series. On the other hand, long-range dependence is characterized by asymptotically hyperbolically decaying autocorrelation function and is thus a global characteristic of the time series. Without imposing any further assumptions, D and H are independent [7]. For self-affine processes, we have $D + H = n + 1$, which implies $D + H = 2$ for a univariate time series. In the context of financial markets, interdependence between D and H implies that local behavior (such as herding behavior) is transmitted into global features of the market (e.g. significant autocorrelations and increased volatility). When we assume that the local behavior is at least partially projected into the global features of the market, then positive long-range dependence (persistence) is connected to a low fractal dimension $D \in (1, 1.5)$ and a higher fractal dimension $D \in (1.5,2)$ is connected to negatively long-range dependent (anti-persistent) processes. If the process is characterized by Hurst exponent close to 0.5 and the fractal dimension close to 1.5, it should have no correlation structure and maximum entropy.

As we are dealing with a time series rather than a texture or some other graphical object, we cannot estimate fractal dimension standardly but we need to use one of the entropies and estimate the information dimension instead. We choose Tsallis' entropy ${}_qH_t = \frac{1-\int g_t^q(x)dx}{q-1}$ with an entropic index q, where $g_t^q(x)$ is a probability density function of increments of process X_t at time t, because it is more appropriate for financial time series – its limiting distribution is q-Gaussian, which is a fat-tailed distribution, and there is no assumption of independence as for Shannon's entropy [20, 21]. Index q is estimated with a method of moments. Fractal dimension D is then estimated as:

$$\hat{D} = \lim_{\varepsilon \to 0} \frac{\dfrac{1-\int g_\varepsilon^{\hat{q}}(x)dx}{\hat{q}-1}}{\log \dfrac{1}{\varepsilon}}$$

where ε is a length of time series partition and $g_\varepsilon(x)$ is a corresponding probability density function. D is assumed to be equal to 1.5 for the efficient market. Obviously, the efficiency index can be generalized to more measures. However, these measures need to be bounded and symmetric around their value for the efficient market.

3. Results

To utilize the developed measure, we analyze 40 world stock indices from the beginning of 2000 until the end of August 2011. The portfolio of indices is geographically broad and covers the North, South and Central America, Western and Eastern Europe, Asia, and Africa (see Table 1). Note that the covered period contains the long-term decreases of stock prices after a bursting of the DotCom bubble as well as a relatively stable growth of the second half of the first decade of the new millennium. The current financial crises are also included. The previously described methods are applied on the close-close logarithmic returns of the daily time series.

Table 1.

Analyzed stock indices, efficiency index, inefficiencies and entropic index *q*.

Ticker	Country	Index	EI	local	global	$\hat{q}$
FTSE	UK	Financial Times Stock Exchange 100 Index	0.2038	0.8111	0.1889	1.3375
SPX	USA/SPX	Standard & Poor's 500 Index	0.2575	0.8847	0.1153	1.3488
DAX	Germany	Deutscher Aktien Index	0.2756	0.9951	0.0049	1.3196
CAC	France	Euronext Paris Bourse Index	0.2856	0.9256	0.0744	1.3264
NIKKEI	Japan	NIKKEI 225 Index	0.3022	0.9875	0.0125	1.3437
SSMI	Switzerland	Swiss Market Index	0.3159	0.9687	0.0313	1.3356
JSE	RSA	Africa All Share Index	0.3314	0.9684	0.0316	1.2921
KFX	Denmark	Copenhagen Stock Exchange Index	0.3334	0.9881	0.0119	1.3306
WIG20	Poland	Warsaw Stock Exchange WIG 20 Index	0.3362	0.9726	0.0274	1.3910
MIBTEL	Italy	Borsa Italiana Index	0.3585	0.9678	0.0322	1.3313
BUX	Hungary	Budapest Stock Exchange Index	0.3664	0.9840	0.0160	1.3361
TA100	Israel	Tel Aviv 100 Index	0.3685	0.9612	0.0388	1.2933
AEX	Netherlands	Amsterdam Exchange Index	0.3805	0.9924	0.0076	1.3347
KS11	South Korea	KOSPI Composite Index	0.3985	0.9943	0.0057	1.3170
IGBM	Spain	Madrid Stock Exchange General Index	0.4065	0.9702	0.0298	1.3779
HSI	Hong-Kong	Hang Seng Index	0.4244	0.9863	0.0137	1.3651
HEX	Finland	OMX Helsinki Index	0.4364	0.9952	0.0048	1.3252
BEL20	Belgium	Euronext Brussels Index	0.4423	0.9882	0.0118	1.3393
BUSP	Brazil	Bovespa Brasil Sao Paulo Stock Exchange Index	0.4473	0.9804	0.0196	1.3164
XU100	Turkey	Instanbul Stock Exchange National 100 Index	0.4484	0.9903	0.0097	1.3171
NASD	USA/NASDAQ	NASDAQ Composite Index	0.4735	0.9926	0.0074	1.3070
BSE	India	Bombay Stock Exchange Index	0.4871	0.9828	0.0172	1.3356
ASE	Greece	Athens Stock Exchange General Index	0.4991	0.9779	0.0221	1.3236
PSE	Philippines	Philippine Stock Exchange Index	0.5011	0.9957	0.0043	1.3930
ATX	Austria	Austrian Traded Index	0.5032	0.9462	0.0538	1.3491
PX50	Czech Rep	Prague Stock Exchange Index	0.5034	0.9866	0.0134	1.3711
IPC	Mexico	Indice de Precios y Cotizaciones	0.5040	0.9840	0.0160	1.3134
TSE	Canada	Toronto Stock Exchange TSE 300 Index	0.5050	0.9803	0.0197	1.3529
SAX	Slovakia	Slovakia Stock Exchange Index	0.5060	0.9762	0.0238	1.3379
STRAITS	Singapore	Straits Times Index	0.5064	0.9749	0.0251	1.3917
DJI	USA/DJI	Dow Jones Industrial Average Index	0.5067	0.9736	0.0264	1.3471
SET	Thailand	Stock Exchange of Thailand Index	0.5075	0.9706	0.0294	1.3825
MERVAL	Argentina	Mercado de Valores Index	0.5079	0.9690	0.0310	1.3300
SSEC	China	Shanghai Composite Index	0.5112	0.9567	0.0433	1.3187
IPSA	Chile	Santiago Stock Exchange Index	0.5123	0.9525	0.0475	1.3598
CSE	Sri Lanka	Chittagong Stock Exchange Index	0.5165	0.9369	0.0631	1.3822
IBC	Venezuela	Caracas Stock Exchange Index	0.5167	0.9365	0.0635	1.3823
JKSE	Indonesia	Jakarta Composite Index	0.5202	0.9240	0.0760	1.3351
KLSE	Malaysia	Bursa Malaysia Index	0.5262	0.9030	0.0970	1.3713
IGRA	Peru	Peru Stock Market Index	0.5493	0.8285	0.1715	1.3583

In Table 1, we present the results for *EI* together with the estimated entropic indices *q*. The most efficient market turns out to be the British FTSE followed by the US S&P500, German DAX, French CAC and Japanese NIKKEI. On the other side of the ranking, the least efficient markets are CSE of Sri Lanka, Venezuelan IBC, Indonesian JKSE, Malaysian KLSE and the least efficient Peruvian IGRA. From geographical point of view, the most efficient markets are the ones of the Western Europe and the USA and on the other side, there are the indices of the Latin America and South/Southeast Asia. As always, there are several exception from this geographical viewpoint such as Brazil and Hong-Kong. Interestingly, even though S&P500 is one of the most efficient stock indices, the other two analyzed US indices (DJI and NASDAQ) are quite inefficient. Moreover, mainly the efficiency/inefficiency of DJI is comparable with the least efficient indices. From the analyzed European indices, the least efficient indices are the ones of Slovakia (the least efficient), the Czech Republic and Austria, while Switzerland, Denmark and Poland are very close to the top ranks. The biggest surprise is the seventh rank of JSE of the Republic of South Africa. However, it is needed to say that approximately from the middle of the ranking to the bottom (except of IGRA), the EIs are very similar (in range from 0.4991 for Greece to 0.5262 for Malaysia) and thus their efficiency is quite comparable. Graphical comparison of indices' relative inefficiencies (ratio of the markets' *EI* with *EI* of the least efficient market, i.e. $\frac{\sqrt{7}}{2}\Big)$, is shown in Figure 1.

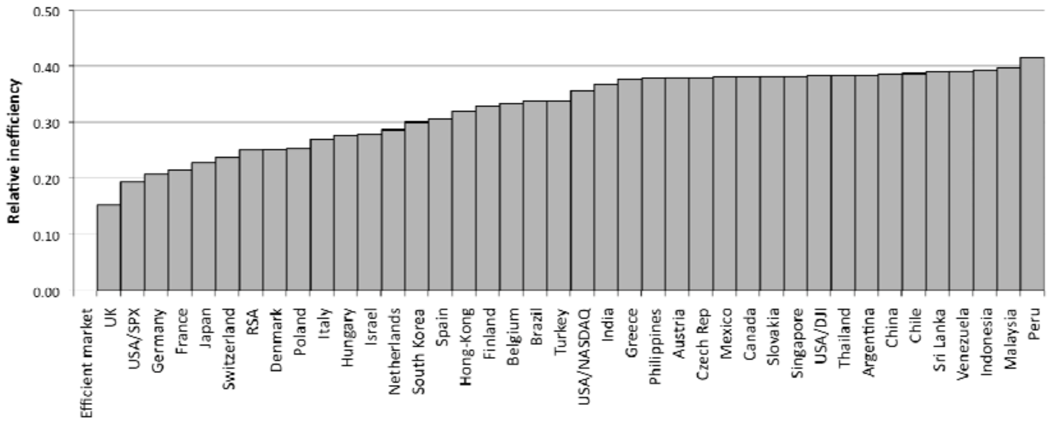

Figure 1. Relative inefficiency of the stock indices.

From an economic point of view, it is also interesting to comment on local and global inefficiencies. As noted earlier, fractal dimension measures local inefficiency whereas Hurst exponent and the first-order autoregression are the measures of global inefficiencies. As the fractal dimension is the only local measure, we can easily write a ratio of local inefficiency as $EI_{local} = \frac{(\hat{D}-1.5)^2}{EI^2}$. A ratio of global inefficiency is then simply $EI_{global} = 1 - EI_{local}$. For financial markets, we assume that if the inefficiency is found, it should be detected as local because a presence of the global inefficiencies would mean an existence of profitable opportunities and potential trading rules, which should vanish very quickly because these would be promptly observed by traders. The results for the ratios between the local and global inefficiencies are summarized in Table 1 as well as in Figure 2. Here, we can see that indeed a strong majority of inefficiency is caused by local characteristics. We observe that the least efficient markets tend to have a higher ratio of global inefficiencies. This is most visible for the least efficient markets. However, we find that three of the most efficient markets (FTSE, SPX and CAC) also have relatively high proportion of global inefficiencies. However, the proportion never surpasses 20% of the total inefficiency.

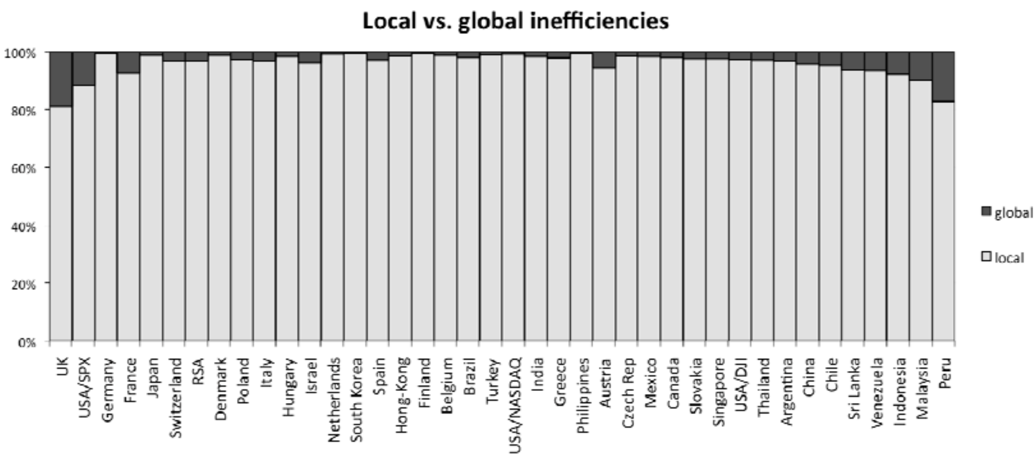

Figure 2. Local vs. global inefficiencies.

4. Conclusions

In summary, we proposed a new measure of the capital market efficiency which can be used to compare several capital markets. Using fractal dimension, short and long-range dependence, we analyzed daily dynamics of 40 stock indices in the period between the beginning of 2000 and the end of August 2011. We found that the most efficient markets are localized in the ”Western” economies (FTSE, S&P500, DAX, CAC and NIKKEI) and the least efficient markets mostly in the Latin America (CSE, IBC, JKSE, KLSE and IGRA). Moreover, we claim that a strong majority of inefficiency can be attributed to local characteristics of the markets (herding and fear factors). The proposed methodology is quite straightforward and can easily be generalized to more measures.

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