MATHEMATICAL MODELING OF WAVE PROPAGATION IN THE FINITE HOMOGENEOUS LINES

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Abstract. *We propose mathematical modeling of wave propagation in the finite homogeneous isotropic lines under the so called "expo functional" influences. Spatial and temporal coordinates vary in arbitrary finite intervals. Analytical study bases on the explicit solution of the corresponding boundary problem in the specific case of the general wave equation caused by the symmetrical differential Maxwell system.*

Keywords and phrases: *mathematical modeling, wave propagation, finite coordinate boundary problem, general wave equation, symmetrical differential Maxwell system.*

1. Introduction

In spite of diversity of computer numerical programs, requirement of effective mathematical modeling and analytical study in modern electrodynamics remains nowadays urgent too [1, 2]. Such scientific direction concerns also phenomena of wave and/or signal propagation in the guide systems including various types of lines. Mathematical modeling here is based on the symmetrical differential Maxwell system [3] that describes the so called "expo functional" influences in terms of those functions' kernels [4]. Solvability criterion of this matrix problem in the meaning of its equivalence to the general wave PDE (partial differential equation) was proved completely in [5]. Such equation contained all scalar components of the unknown electromagnetic field intensities, and investigated medium was isotropic linear and homogeneous. Moreover, only ordinary, non generalized functions were taken into account.

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Though the given general spatial case of (x, y, z, t) was very important, specific statement of (x, t) remains practically required as well. It is quite natural, since electrodynamics processes are considered here not in the waveguides, but in the lines, whose mathematical representation can not be got by direct simple reduction of (x, y, z, t) to (x, t). Confirmation of this fact is shown in the next section when the symmetrical Maxwell system appears.

Turning again to the aforesaid solvability criterion, it should be noted that its existence allows formulating relevant electrodynamics phenomena analytically in terms of the boundary problems regarding the general scalar wave PDE. It is clear that such approach is incomparably easier than dealing with the original matrix/vector version. Hence, quite a lot of respective electrodynamics processes can be studied effectively using those boundary statements as their mathematical simulation. Thus the wave propagation under "expo functional" influences for the semi-infinite isotropic homogeneous lines and arbitrary finite time gaps was explicitly investigated in [6].

Unfortunately, rather important case of the same problem for arbitrary finite lines was not considered yet. That is why the goal of the present paper is mathematical modeling and constructive study of wave propagation in arbitrary isotropic homogeneous finite lines with various finite time gaps.

2. Mathematical modeling in terms of Maxwell system

First of all, let the symmetrical differential Maxwell system be given for homogeneous isotropic linear media in the presence of "expo functional" influences when their kernels are studied:

$$\begin{cases} \mathbf{rot}\vec{H} = (\sigma \pm \lambda\varepsilon_a)\vec{E} + \varepsilon_a \partial_0 \vec{E} + \vec{j}^{OS} \\ -\mathbf{rot}\vec{E} = (r \pm \lambda\mu_a)\vec{H} + \mu_a \partial_0 \vec{H} + \vec{e}^{OS}. \end{cases} \quad (1)$$

In (1): the unknown vector functions $\vec{E}, \vec{H} = \vec{E}, \vec{H}(x, y, z, t)$ with scalar components $E_k, H_k = E_k, H_k(x, y, z, t)\ (k = \overline{1,3})$ represent electric and magnetic field intensities; positive constants σ, μ_a, ε_a are the specific conductivity, absolute and dielectric permeability of the medium respectively; vector functions $\vec{j}^{OS}, \vec{e}^{OS} = \vec{j}^{OS}, \vec{e}^{OS}(x, y, z, t)$ are known and describe the outside current sources and intensities. Their scalar components look like $j_k^{OS}, e_k^{OS} = j_k^{OS}, e_k^{OS}(x, y, z, t)$ $(k = \overline{1,3})$. Parameter of the signal exciting medium is $\lambda = const > 0$. Sign reversal in front of λ corresponds to the absorption of signal for "+" and activity of the medium for the "-".Theoretical constant $r > 0$ is assumed only at the current research stage. It is responsible for the system symmetry and simplification of the further mathematical computation. At the end, it can be omitted not influencing the original problem statement and final numerical calculation. Partial differential temporal operator and classical rotor definition look like:

$$\partial_0 = \frac{\partial}{\partial t}, \ \mathbf{rot} = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_1 & \partial_2 & \partial_3 \\ F_{i1} & F_{i2} & F_{i3} \end{bmatrix}, \text{ where } \partial_1 = \frac{\partial}{\partial x}, \ \partial_2 = \frac{\partial}{\partial y}, \ \partial_3 = \frac{\partial}{\partial z}.$$

Functions $F_{ik} = F_{ik}(x, y, z, t)$ $(i = 1,2;\ k = \overline{1,3})$ designate scalar components of the appropriate vector electromagnetic field intensities $\vec{F}_1, \vec{F}_2 = = \vec{F}_1, \vec{F}_2(x, y, z, t)$; $\vec{F}_1, \vec{F}_2 = \vec{E}, \vec{H}$.

Mathematical simulation of the specific version of (1) in the case of wave propagation through the excited homogeneous isotropic lines with expo functional influences, is written below:

$$\begin{cases} \partial_1 H = (\sigma + \varepsilon_a \partial_0^*)E + j^{OS} \\ -\partial_1 E = (r + \mu_a \partial_0^*)H + e^{OS}. \end{cases} \tag{2}$$

In (2), all symbols are of the same meaning as it was introduced in (1), and new partial differential operator looks like $\partial_0^* = \partial_0 \pm \lambda$. Here, in comparison with (1), electromagnetic field functions $E, H,\ j^{OS}, e^{OS}$ are not vectors, but scalars and belong to one and the same ordinary differentiated functional class that is determined by the structure of (2). Later, this class can be modified taking into account the particular boundary problem conditions.

As it was said earlier in the section 1, system (2) is really the specific case of (1). It is completely explicable because one-dimensional rotor operation does not exist. So, the direct pure reduction of (1) to (2) is impossible without detailed additional study in the suggested particular statement of (x, t).

3. Preliminary results

Rewriting (2) in the matrix form we get:

$$MF = f, \; M = \begin{bmatrix} -C & \partial_1 \\ -\partial_1 & -D \end{bmatrix}, \tag{3}$$

where: $C = \sigma + \varepsilon_a \partial_0^*$; $D = r + \mu_a \partial_0^*$ – are the partial differential operators;

$F_1 = E,\ F_2 = H;\quad f_1 = j^{OS},\ f_2 = e^{OS};\quad F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$ – are the aforesaid electromagnetic field functions.

Application to (2) of the inverse matrix operator M^{-1} regarding (3):

$$M^{-1} = (\det M)^{-1} \begin{bmatrix} -D & -\partial_1 \\ \partial_1 & -C \end{bmatrix}, \; \det M = \partial_1^2 + CD, \tag{4}$$

reduces (2) to the general wave PDE [6]

$$(\tilde{\partial}_0^2 + \partial_1^2)F = \tilde{f}. \tag{5}$$

In (4), (5): $\det M$ – is the determinant of (3); $(\det \mathrm{M})^{-1}$ – is the inverse scalar operator with respect to $\det M$:

$\tilde{\partial}_0^2 = CD = (\sigma + \varepsilon_a \partial_0^*)(r + \mu_a \partial_0^*) = \varepsilon_a \mu_a \left(\partial_0^*\right)^2 + \left(\sigma\mu_a + r\varepsilon_a\right)\partial_0^* + \sigma r$ – is the operator differential polynomial:

$\tilde{f} = \begin{bmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{bmatrix} = \begin{bmatrix} -Df_1 - \partial_1 f_2 \\ \partial_1 f_1 - Cf_2 \end{bmatrix}$ – are the known functions, and operator "power" is understood in its usual meaning as the consecutive operator action.

Inverse application of (3) to (5) transforms this wave scalar equation backwards to (2). Analysis of those conditions while $\det M \neq 0$, carries out investigation of existence of (4) using square operator polynomial $\tilde{\partial}_0^2 + \partial_1^2$ in terms of ∂_0^*. Thus, the required necessity and sufficiency of the inverse matrix operator construction are proved, and the following theorem is true:

Solvability criterion of (2): *the specific case (2) of symmetrical differential Maxwell system (1) is solved explicitly in the meaning of its equivalence to the general scalar wave PDE (5) iff conditions:*

$$\left[\begin{array}{l} \partial_1^2 > \left(\frac{1}{2}(\sigma\sqrt{\mu_a/\varepsilon_a} - r\sqrt{\varepsilon_a/\mu_a})\right)^2 \\ \left\{\begin{array}{l} \partial_1^2 \le \left(\frac{1}{2}(\sigma\sqrt{\mu_a/\varepsilon_a} - r\sqrt{\varepsilon_a/\mu_a})\right)^2 \\ \partial_0 \ne \mp\lambda - \frac{1}{2}\left((\sigma/\varepsilon_a + r/\mu_a) \pm \sqrt{(\sigma/\varepsilon_a - r/\mu_a)^2 - 4\partial_1^2/(\mu_a\varepsilon_a)}\right) \end{array}\right. \end{array}\right. \tag{6}$$

are valid, and only ordinary classical, not generalized functions are taken into account.

Sign reversal in front of λ in (6) is independent of the sign value near the square root.

Here, we should like to remind that (2), as the particular case of symmetrical differential Maxwell system (1), has to be studied separately since the one-dimensional analogy of the classical **rot** operation does not exist. Moreover, the main virtue of the given criterion is essential simplification simulating mathematically signal and/or wave propagation in radio engineering and telecommunications. Namely, relevant boundary problems describing respective phenomena can be expressed in terms of PDE (5). The last fact cannot be even compared with the original system (2) by its new obvious scalar version. It is quite natural, because in general, corresponding correct boundary problem statement appears completely ambiguous in the matrix / PDEs' system form.

4. Boundary problem solution

Moving to the concrete mathematical modeling of the wave propagation in the finite homogeneous isotropic lines with arbitrary finite time gaps, we introduce the relevant boundary problem that is written below:

$$\begin{cases} (\tilde{\partial}_0^2 + \partial_1^2)F = \tilde{f}, \ \ x \in [0,a], t \in [0,T]; \\ F(x,0) = g_1(x); \\ F(x,T) = g_2(x); \\ F(0,t) = g_3^*(t); \\ F(a,t) = g_4^*(t). \end{cases} \qquad \begin{array}{l} 0 \_\_\_\_\_\_ a \_\_\_ x \_\_\_ \rightarrow \\ t \in [0,T] \end{array} \tag{7}$$

In (7): $g_j(x)$ $(j=1,2)$, $g_k^*(t)$ $(k=3,4)$ – are the known functions existed in the corresponding intervals and required functional classes.

Explicit solution of (7) is done by means of the Fourier finite integral sine transform [7] application to the spatial variable x along the interval $[0,a]$ and considering temporal argument t as the main one. Hence, integrating by parts twice:

$$\frac{\pi}{a}\int_0^a \partial_1^2 F(x,t)\sin\left(n\frac{\pi}{a}x\right)\mathrm{d}\,x = -\frac{\pi^3}{a^3}n^2F_n(t)+\frac{\pi^2}{a^2}n((-1)^{n+1}F(a,t)+F(0,t)) =$$

$$=\left(\frac{\pi}{a}\right)^2 n\left(-\frac{\pi}{a}nF_n(t)+(-1)^{n+1}g_4^*(t)+g_3^*(t)\right)$$

and using this last expression we reduce (7) to the boundary problem with respect to the Fourier integral sine transforms. Namely:

$$\begin{cases} \left(\dfrac{\mathrm{d}^2}{\mathrm{d}\,t^2}+a\dfrac{\mathrm{d}}{\mathrm{d}\,t}+b\right)F_n(t) = f_n^*(t),\ \ t\in[0,T]; \\ F_n(0)=g_{1n}; \\ F_n(T)=g_{2n}, \end{cases} \tag{8}$$

where:

$$a=\sigma/\varepsilon_a + r/\mu_a \pm 2\lambda\,, b=\lambda^2 \pm \lambda\,(\sigma/\varepsilon_a + r/\mu_a)+\left(\sigma r-\left(\frac{\pi}{a}\right)^3 n^2\right)\Big/(\mu_a\varepsilon_a)$$

– are the constant coefficients;

$$g_{jn}=\frac{\pi}{a}\int_0^a g_j(x)\sin\left(n\frac{\pi}{a}x\right)\mathrm{d}\,x\ \ (j=1,\,2);$$

$$F_n=F_n(t)=\frac{\pi}{a}\int_0^a F(x,t)\sin\left(n\frac{\pi}{a}x\right)\mathrm{d}\,x;\ \tilde{f}_n(t)=\frac{\pi}{a}\int_0^a \tilde{f}(x,t)\sin\left(n\frac{\pi}{a}x\right)\mathrm{d}\,x;$$

$f_n^* = f_n^*(t)=\left(\tilde{f}_n(t)-\left(\frac{\pi}{a}\right)^2(-1)^{n+1}g_4^*(t)+g_3^*(t)\right)\Big/(\mu_a\varepsilon_a)$ – are the corresponding Fourier sine transforms whose lower index n means transformation operation.

Solving linear inhomogeneous ODE (ordinary differential equation) with constant coefficients from (8) by means of the well-known technique [8] we get the sought for transforms of electromagnetic intensities as the general solution of this mentioned equation.

Really, performance equation of ODE from (8) is written below:

$$\omega^2 + a\omega + b = 0. \qquad (9)$$

The roots ω_j $(j = 1,2)$ and discriminant D of (9) look like:

$$\sqrt{D} = \omega_1 - \omega_2,\ \omega_j = \left(-a + (-1)^{j+1}\sqrt{D} \right)/2 \ \ (j = 1,\ 2),$$

$$D = (\sigma/\varepsilon_a - r/\mu_a)^2 + 4\left(\frac{\pi}{a}\right)^3 n^2 /(\mu_a\varepsilon_a) > 0. \qquad (10)$$

Turning to (10), it is easy to notice that both roots are real. Therefore, the general solution $F_{0n} = F_{0n}(t)$ of the homogeneous ODE regarding original one from (8) has the following structure:

$$F_{0n} = \sum_{j=1}^{2} C_j^* \,\mathrm{e}^{\omega_j t},\ \forall C_j^* = \mathrm{const} \in \mathbf{R}. \qquad (11)$$

It is well known [8] that the general solution $F_n = F_n(t)$ of the original ODE from (8) is the sum of (11) and particular solution $F_{pn} = F_{pn}(t)$ of the same inhomogeneous ODE from (8):

$$F_n = F_{0n} + F_{pn}. \qquad (12)$$

Basing on the classical theory of linear ODEs [8] function F_{pn} is looking for as the functional sum:

$$F_{pn} = \sum_{j=1}^{2} C_j(t)\,\mathrm{e}^{\omega_j t},\ \ C_j(t) - ? \qquad (13)$$

The unknowns $C_j(t)$ $(j = 1,2)$ in (13) are found from the differential system [8]:

$$\begin{cases} \sum_{j=1}^{2} C_j'(t)\,\mathrm{e}^{\omega_j t} = 0 \\ \sum_{j=1}^{2} C_j'(t)\omega_j\,\mathrm{e}^{\omega_j t} = f_n^*,\ \ C_j'(t) = \dfrac{\mathrm{d}\,C_j(t)}{\mathrm{d}\,t} \quad j = 1,2 \end{cases}$$

and are described by the general formula:

$$C_j(t) = \frac{(-1)^{j+1}}{\sqrt{D}} \int \mathrm{e}^{-\omega_j t}\ f_n^*\,\mathrm{d}\,t = \frac{(-1)^{j+1}}{\sqrt{D}} s_j(t) \quad j = 1,2. \qquad (14)$$

Substitution of (14) into (13) together with (12) and (11) gives:

$$F_n = \sum_{j=1}^{2}\left(C_j^* + \frac{(-1)^{j+1}}{\sqrt{D}} s_j(t) \right)\exp(\omega_j t),\ \forall C_j^* = \text{const} \in \mathbf{R}. \qquad (15)$$

Implementation of the initial conditions from (8) allows determining C_j^* $(j = 1,2)$ from the following system:

$$\begin{cases} \sum_{j=1}^{2} C_j^* = g_{5n}^*,\ g_{5n}^* = g_{1n} + \frac{1}{\sqrt{D}} \sum_{j=1}^{2} (-1)^j s_j(0) \\ \sum_{j=1}^{2} C_j^* \mathrm{e}^{\omega_j T} = g_{6n}^*,\ g_{6n}^* = g_{2n} + \frac{1}{\sqrt{D}} \sum_{j=1}^{2} (-1)^j s_j(T)\,\mathrm{e}^{\omega_j T} \end{cases}. \qquad (16)$$

The known functions $s_j(t) = \int \mathrm{e}^{-\omega_j t}\ f_n^*(t)\,\mathrm{d}t$ $(j = 1, 2)$ at first appeared in (14), and required constants C_j^* $(j = 1,2)$ are found from (16):

$$C_j^* = \frac{(-1)^{j+1}(g_{6n}^* - g_{5n}^* \exp(\omega_{j+(-)^{j+1}} T))}{(\exp(\omega_1 T) - \exp(\omega_2 T))},\ (j = 1,2). \qquad (17)$$

Substitution of (17) into (15) gives the required solution of boundary problem (8) in terms of Fourier transforms. Thus, we get:

$$F_n = \sum_{j=1}^{2} (-1)^{j+1} \left(\frac{g_{6n}^* - g_{5n}^* \exp(\omega_{j+(-1)^{j+1}} T)}{\exp(\omega_1 T) - \exp(\omega_2 T)} + \frac{s_j(t)}{\sqrt{D}} \right) \exp(\omega_j t), \qquad (18)$$

and functions g_{5n}^*, g_{6n}^* are denoted in (16).

Application of the inverse Fourier finite sine transform [7] $\frac{2a}{\pi^2} \sum_{n=1}^{\infty} \sin\left(n \frac{\pi}{a} x \right)$ to (18) leads to the desired solution of the original boundary problem (7), i.e.:

$$F(x,t) = \frac{2a}{\pi^2} \sum_{n=1}^{\infty} F_n \sin\left(n \frac{\pi}{a} x \right) =$$

$$\frac{2a}{\pi^2} \sum_{n=1}^{\infty} \sum_{j=1}^{2} (-1)^{j+1} \left(\frac{g_{6n}^* - g_{5n}^* \exp(\omega_{j+(-1)^{j+1}} T)}{\exp(\omega_1 T) - \exp(\omega_2 T)} + \frac{s_j(t)}{\sqrt{D}} \right) \exp(\omega_j t)\ \sin\left(n \frac{\pi}{a} x \right). \qquad (19)$$

The last final formula represents constructive result of the original problem statement, and it means that the goal of the present paper is completely attained.

Furthermore, the next identity:

$$\mathrm{e}^{\omega_1 T} - \mathrm{e}^{\omega_2 T} = \mathrm{e}^{-\frac{a}{2}T}\, 2\,\mathrm{sh}\left(\frac{\sqrt{D}}{2}T\right)$$

reduces (19) to the equivalent expression:

$$F(x,t) =$$

$$= \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \sum_{j=1}^{2} (-1)^{j+1} \left(\frac{g_{6n}^{*}\, \mathrm{e}^{\frac{a}{2}T} - g_{5n}^{*}\, \mathrm{e}^{(-1)^{j+1}\frac{\sqrt{D}}{2}T}}{2\,\mathrm{sh}\left(\frac{\sqrt{D}}{2}T\right)} + \frac{s_j(t)}{\sqrt{D}} \right) \mathrm{e}^{\omega_j t} \sin\left(n\frac{\pi}{a}x\right). \quad (20)$$

Further analysis of absolute functional series' convergence [9] in (19) = (20) with respect to $x \in [0, a]$ is supported by the following inequality:

$$\frac{2a}{\pi^2} \sum_{n=1}^{\infty} \left| F_n \sin\left(n\frac{\pi}{a}x\right) \right| \le \frac{2a}{\pi^2} \sum_{n=1}^{\infty} |F_n| =$$

$$= \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \left| \sum_{j=1}^{2} (-1)^{j+1} \left(\frac{g_{6n}^{*}\, \mathrm{e}^{\frac{a}{2}T} - g_{5n}^{*}\, \mathrm{e}^{(-1)^{j+1}\frac{\sqrt{D}}{2}T}}{2\,\mathrm{sh}\left(\frac{\sqrt{D}}{2}T\right)} + \frac{s_j(t)}{\sqrt{D}} \right) \right| \mathrm{e}^{\omega_j t}.$$

The last depends on those coordinate, medium's parameters and given functions that are responsible for description of studied wave propagation.

5. Concluding remarks

First of all, coming to the shortcomings of the given paper, the lack of concrete numerical computation can be noticed. It means that just at this moment, present investigation has only theoretical analytical research tendency.

Nevertheless, explicit solution of (7) that is provided by the last expressions (19), (20) in Section 4, is easily calculated for various numerical values of $T, a, \sigma, \varepsilon_a, \mu_a, \lambda, r$ and corresponding functions $g_j(x)$ $(j = 1,2)$, $g_k^*(t)$ $(k = 3,4)$, j^{OS}, e^{OS} from formulae (7), (2). So, it is clear now that the next computing and experimental verification is not only supported, but essentially simplified already because of (19) = (20) as the explicit solution of the original problem (7). Hence, the further numerical results are the corollaries of the suggested exact formulas (20), (19), (16), (8), and are obtained easier in comparison of existing approximate computing procedures [1, 2].

Taking into account all aforesaid information, it can be mildly concluded that proposed here mathematical analytic method has its own virtues basing upon obvious generalization to the bigger dimensional systems of PDEs. The latter are used as mathematical simulation not only in electromagnetic field theory, but in other various aspects of applied / industrial problems.

Moreover, turning to [6], it is easy to find apparent similarity between (20) and final result from [6] when $x \in [0, \infty)$ and t remains the same as in the present paper. Actually, the main difference bases on the interval variation of x and consists of the finite integral sine Fourier application instead of the continuous one in [6]. Detailed theoretical and numerical analysis of interrelations between those two cases proposed in [6] and in the present paper is planned to be studied in the nearest future. This challenge looks interesting and important as from the analytical, as from the applied engineering viewpoints.

At the very end, clarifying the last mentioned information, the simple table is proposed here. It shows evident interconnections between objects in the given article (they are located on the left) and their analogies from [6] (they are placed on the right). Namely, index of Fourier transform $n \mapsto$ becomes index of Fourier transform α; $g_3^*(t) \mapsto g_3(t)$; $g_4(t) \mapsto 0$; $g_{6n}^*, g_{5n}^* \mapsto g_{5\alpha}, g_{4\alpha}$; in D instead of $\left(\frac{\pi}{a}\right)^3 n^2 \mapsto \alpha$ appears:

$E, H \mapsto H, E$;

$$f_n^* = f_n^*(t) = \left(\tilde{f}_n(t) - \left(\frac{\pi}{a}\right)^2 ((-1)^{n+1} g_4^*(t) + g_3^*(t)) \right) \Big/ (\mu_a \varepsilon_a) \mapsto$$

$$f_\alpha^* = f_\alpha^*(t) = -(\tilde{f}_\alpha(t) + \alpha g_3(t)) / (\mu_a \varepsilon_a);$$

$\widetilde{f}_1 = \widetilde{f}_1(x,t) = -Dj^{OS} - \partial_1 e^{OS} \mapsto \widetilde{f}_1 = \widetilde{f}_1(x,t) = -\partial_1 j^{OS} + Ce^{OS}$;

$\widetilde{f}_2 = \widetilde{f}_2(x,t) = \partial_1 j^{OS} - Ce^{OS} \mapsto \widetilde{f}_2 = \widetilde{f}_2(x,t) = \partial_1 e^{OS} + Dj^{OS}$, and all other symbols remain the same in both cases.

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