# COMPETITION BETWEEN TWO INTERDEPENDENT BINARY OPINION NETWORKS: THE ROLE OF OPPORTUNISTS AND CONTRARIANS

Karl E. KÜRTEN*

***Abstract.*** *We study a binary model for the process of opinion formation among agents having the choice between two products. The products A and B are represented by two networks specified by randomly chosen infra couplings within the networks and interdependent inhibitory couplings between the two networks which compete for the choice of the two products. The interactions within the two networks are based on the familiar local majority/minority rules of behaviour. Due multistability, a subtle interplay between opportunists, contrarians leads to dynamical phase transitions which abruptly can change the outcome of the acceptance of one of the two products. "Tie" phenomena stemming from the coexistence of opportunists and contrarians, reminiscent of various recent hung elections, where the systems turn out to be highly unstable with respect to minor local perturbations in the original uncoupled systems, can be successfully overcome by the choice of the initial conditions.*

***Keywords:*** *social networks, opinion spreading, dynamical phase transitions, majority/minority rule, opportunistic-contrarian behaviour.*

## 1. Introduction

The theoretical study of discrete choice has long been a subject of interest in social and economic sciences. During the last decades, after social interactions, had first been introduced for sociological problems by social scientists, theoretical physicists started to apply methods commonly used in the area of statistical physics to social phenomena such as opinion formation and economic dynamics [1, 2, 3, 4]. Indeed, there is an intimate relationship between ferro- and anti-ferromagnetic models and collective decision making processes. The magnetic moment of a particle can be

* Faculty of Physics, Vienna University, 5, Boltzmanngasse, A-1090 Vienna, Austria, Department of Physics, Loughborough University, Leicestershire, LE11 3TU, UK, e-mail: karl.kuerten@univie.ac.at.

identified with the choice of an agent, spin up could be identified with YES and spin down with NO, respectively. Social interactions of agents are then naturally described by the process of the tendency to align or disalign. Up to now, mostly single isolated networks have been studied in this context. However, social real-world infrastructures usually interact with each other, and their dynamical behaviour depends crucially on the dynamics of various other networks. This gives rise to inter- couplings between networks, which are usually not isolated at all. We consider a system consisting of *N* agents who discuss the acceptance or refusal of a two competing choices *A* and *B*, which are interdependent and exclude each other.

Indeed, almost all real world networks are interconnected and non-trivially depend on each other, since there is a crucial need for mutual activation or inhibition. To give an example, various infrastructures such as water supply, transportation, fuel and power stations are coupled together and depend on each other such that even localized random damage in one network may lead to failures in the interdependent networks and can produce cascades of failures [3]. In this model, each agent *i* is assigned *K* randomly chosen agents who try to convince agent *i* to make a decision, in favour or against with respect to product *A* or product *B*. The dynamical decision making process is based on the majority- and minority rule, where opportunists adopt the opinion of the majority, where the contrarians adopt the opinion of the minority with respect to their local neighbourhood. The model thus may also describe how agents from different cultures try to convince each other, in either their or in the opposite direction.

In particular, we study the effects of mixed interactions between opportunists and contrarians based on only two simple behaviour schemes expressed by simple threshold rules which in the mixing case are able to generate rather complex behaviour. One focus of this study is the determination of critical network parameters, where the network dynamics undergo transitions from the existence of two attractors, the YES and NO attractor, to one unique stable attractor [5, 6].

## 2. Two interdependent networks

The system is made up of two coupled networks *A* and *B*, each of which consists of *N* interacting agents described by binary variables $x_i^A(t)$

and $x_i^B(t)$, respectively. Within network $A$ and network $B$ the nodes are randomly connected via intra couplings $c_{ij}$, where each agent $i$ is influenced by exactly $K$ randomly chosen agents $j_1(i), \ldots, j_K(i)$ of its own network. We assume further that each agent $i$ of network $A$ interacts only with one and only one single agent $j_i$ of network $B$. Agent $j_i$ can be chosen at random or could be its counterpart in the competing network $(j_i = i)$. The inter coupling between the two networks can be chosen bi-directional in contrast to the intra-links within the individual networks which are usually uni-directional. In principle, network $A$ and network $B$ could be identical such that agent $i$ votes for the choice $A$ and the choice $B$ in the two networks [2].

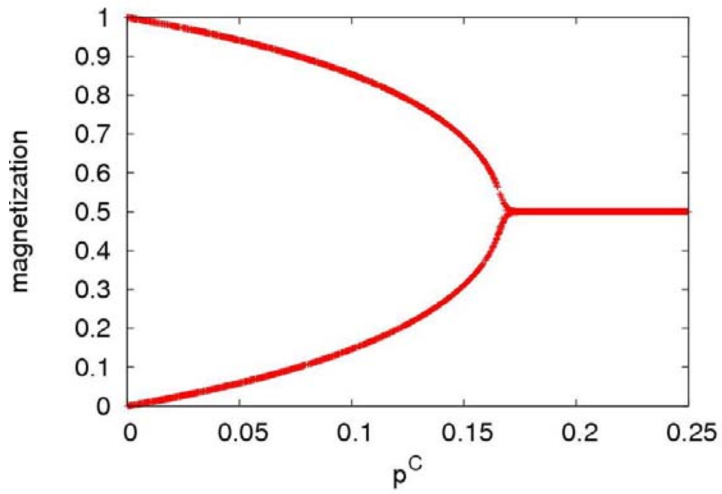


**Figure 1.** Asymptotic behaviour for an uncoupled network.

Following the lines of the majority/minority principle the intra couplings are chosen as $c_{ij} = 1$ with probability $p^O$ and $c_{ij} = -1$ with probability $p^C = 1 - p^O$ for all $j = 1, \ldots, N$, where $i$ is fixed such that all incoming stimuli to unit $i$ are either excitatory (ferro-) or inhibitory (anti-ferromagnetic). Note that we restrict $p^C$ to small values, since the fraction of contrarians is usually small and they are by far in the minority. The time dependent variables $x_i^A(t)$ and $x_i^B(t)$ define the YES (1) and NO (0) state of agent $i$ for choice $A$ and for choice $B$, respectively. For simplicity we will further concentrate on $K$ odd where always exists a majority in favour of one of the two choices. Opportunists follow the majority opinion in their local neighbourhood and obey the local majority rule, while the contrarians, who always adopt the opinion opposite of that of the majority, obey the local minority rule.

Starting with initial values $x_i^A(0)$ and $x_i^B(0)$ at time $t = 0$ we define the dynamical time evolution in terms of a linear generalized threshold rule widely used in models of neural and genetic networks [7, 8]:

$$x_i^A(t+1) = (1 - x_i^B(t))\Theta\left(\sum_{(j)} c_{ij}^A\left(x_j^A(t) - \frac{K}{2}\right)\right) \tag{1}$$

and

$$x_i^B(t+1) = (1 - x_i^A(t))\Theta\left(\sum_{(j)} c_{ij}^B\left(x_j^B(t) - \frac{K}{2}\right)\right) \tag{2}$$

where the threshold value $\frac{K}{2}$ is adapted to the majority/minority rule. The inter-coupling product terms $(1 - x_i^B(t))$ and $(1 - x_i^A(t))$ imply that an agent cannot vote for both choices at the same time. The index (*j*) runs over *K* randomly chosen neighbours of agent *i*. The theta function is specified by $\Theta(x) = 1$ for $x \geq 0$, and $\Theta(x) = 0$ for $x < 0$. The sum in the theta function represents the individual stimulus of unit *i* received from the elements of its own network.

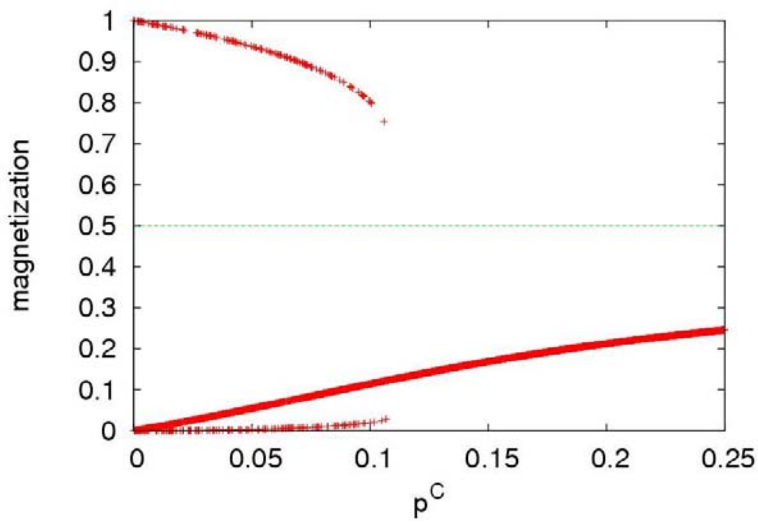


**Figure 2.** Asymptotic behaviour for two coupled networks.

A suitable order parameter, the public opinion at time *t*, which defines the degree of acceptance of the YES or NO state, can be defined by the total magnetizations:

$$m^A(t) = \frac{1}{N}\sum_{j=1}^{N} x_j^A(t) \quad \text{and} \quad m^B(t) = \frac{1}{N}\sum_{j=1}^{N} x_j^B(t) \tag{3}$$

the average magnetization of the two networks.

## 3. Analytical results

In the thermodynamic limit of asymptotically large *N*, statistical predictions for the time evolution of the magnetization $m(t)$ Eq. (4) and Eq.(5) can be derived [5]. The time evolution of the magnetization or public opinion with respect to the YES state $m(t+1)$ is a linear superposition of all contributions to the YES state. Eventually it takes the form:

$$m_A(t+1) = (1 - m_B(t))\,((1-2p^C)\,(3m_A^2(t) - 2m_A^3(t)) + p^C) \tag{4}$$

$$m_B(t+1) = (1 - m_A(t))\,((1-2p^C)\,(3m_B^2(t) - 2m_B^3(t)) + p^C). \tag{5}$$

Figure 1 (upper) depicts the asymptotic magnetization *m** for an uncoupled network as a function of the concentration of the contrarians $p^C$. If only opportunists are present in the uncoupled model ($p^C = 0$), the competing states YES (1) or NO (0) are monotonically approached after a short transient, i.e. the only stable states are the fully ordered ones. With increasing values of $p^C$ we have two equally probable attractors which define a state of high and low demand. They are reached depending on which product is favoured in the initial condition. For $p^C = \frac{1}{6}$ we have a backward bifurcation and for $p > p^C$ the tie state $m^* = \frac{1}{2}$ is reached independent of the initial condition. This second order phase transition, where one spin flip implies a cascade of flips of other agents leads to a chaotic state. Note however, that the macroscopic state, the magnetization $m^* = \frac{1}{2}$ is a stable attractor. We recover the familiar uncoupled network if we set $x_i^B = 0$ in Eq. (4) and simply ignore Eq. (5). The tie phenomenon, a pure symmetry effect, since the rules are symmetric, was first reported by Galam in his original model [4, 5].

For the coupled system, depicted in Figure 2, the situation is similar. Only for $p^C = 0$ we have polarisation, i.e. fixed points $(m_A^*, m_B^*) = (1,0), (m_A^*, m_B^*) = (0,1),$ and $(m_A^*, m_B^*) = (0,0)$ which are all stable.

For $p^C \neq 0,$ in marked contrast to the uncoupled model we have three potential states of equilibrium of high, middle and low demand, however the symmetry is. The two fixed points, which signal a more or less clear decision for product *A* or *B*, are not symmetric any more. In analogy to the uncoupled system there is a critical $p^C \approx 0{,}107,$ where the two fixed points become unstable. However in the coupled case the phase transition is first order and two branches bifurcate discontinuously into a unique fixed point $(m_A^*, m_B^*)$ with $(m_A^* = m_B^*).$ The third fixed point with $(m_A^* = m_B^*)$ signals a tie situation at a rather low level of acceptance for both products. It is stable over the whole range $p > p_C$ and could be interpreted as an important fraction of non-voters. Provided that the number of the initial acceptance of product A and B are both high the two networks compete strongly for the votes and they mutually reduce their acceptance, since the agent is not allowed to vote for *A* and *B* simultaneously.

The stable and unstable solutions of Eq. (4) can be solved graphically by the following two curves crossing in the $(m_A^*, m_B^*)$ – plane defined by:

$$m_B = f_A(m_A) = 1 - \frac{m_A}{(1-2p^C)(3m_A^2 - 2m_A^3) + p^C} \tag{6}$$

$$m_A = f_B(m_B) = 1 - \frac{m_B}{(1-2p^C)(3m_B^2 - 2m_B^3) + p^C} \tag{7}$$

where the function $f_B(m_A)$ is the inverse of $f_A(m_A).$ The long time behaviour is governed by the basins of attraction of the stable fixed points of the system presented in Figure 3 and Figure 4. Preferentially, initial conditions, with $m_A$ small and $m_B$ large, guarantee a clear victory for *A* or *B*. The boundary line which separates the basins of attraction of the tie situation and the two states of high and lower acceptance can be approximated by the superelliptical curve specified by the equation:

$$\left(\frac{m_A}{a}\right)^{\alpha} + \left(\frac{m_B - 1}{b}\right)^{\alpha} = 1 \tag{8}$$

with $\alpha < 2$, where the unshaded areas within the superelliptic shape specify the $(m_A, m_B)$ space, where we have $m_A^* \neq m_B^*$. In this areas the product with more favourable initial condition for the YES always wins. In the shaded area the time evolution ends up with $(m_A^* = m_B^*)$, the tie situation.

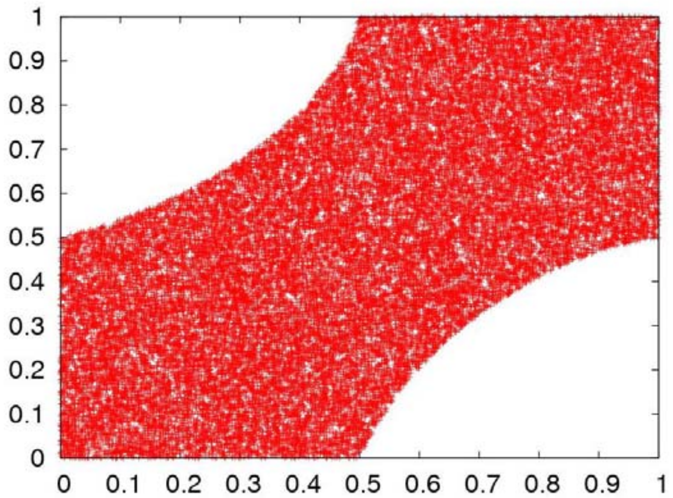


**Figure 3.** Basin of attraction in the $(m_A, m_B)$ – plane for $p_c^O = 0$ with $\alpha = 1.46$, $b = 0.5$, $\alpha = 0.5$.

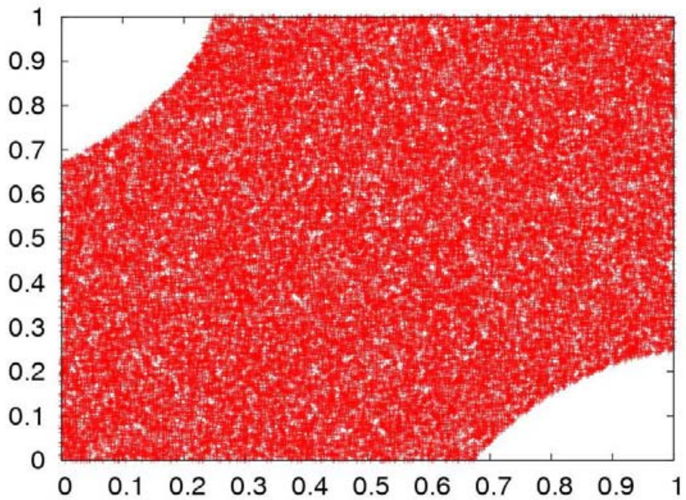


**Figure 4.** Basin of attraction in the $(m_A, m_B)$ – plane for the near critical $p^O = 0.106$ with $\alpha = 1.4$, $b = 0.325$, $\alpha = 0.248$.

## 4. Summary

We have presented two coupled networks which compete for the vote of two choices via inhibitory interdependent interactions. The agents are described by two prototypes of opposite characters in order to study the subtle balance of opportunistic and contrarian behaviour in the dynamics of opinion spreading. Due to multistability it is eventually only the initial condition which decides about winning, loosing, or the tie situation which prevails. With increasing fraction of the contrarians the basin of attraction for a clear victory of one of the two choices becomes smaller and eventually vanishes at a critical concentration of the contrarians $p_c^C \approx 0.107$, where, independent of the initial conditions, we have a tie situation with a relatively low rate of votes for the acceptance state. At lower concentrations of the contrarians we find two stable fixed point configurations signalling a more or less clear YES or NO majority. Beyond the critical point $p_c^C$ we find a unique stable fixed point which corresponds to the tie situation independent of the initial condition.

## 5. Acknowledgments

The author thanks S. Galam and F. V. Kusmartsev for various valuable discussions.

REFERENCES

[1] C. Castellano, S. Fortunato and V. Loreto, *Statistical Physics of Social Dynamics, Reviews of Modern Physics*, 81:591-646 (2009).

[2] A. H. Kunzhao A. Baronchelli, and G. Bianconi EPL 102 16002 (2013).

[3] J. Gao, S. V. Buldyrev, H. E. Stanley, and S. Havlin, *Nature physics*, **8,** 2012, 40-48.

[4] S. Galam, Intern. J. of Modern Phys. **C 19, 3,** 409-440 (2007).

[5] K. E. Kürten, Int. J. Mod. Phys. **B 22,** 25-26, 4674-4683 (2008).

[6] S. Galam, Eur. Phys. J. B **25,** 403-406 (2002).

[7] K. E. Kürten, Phys. Lett. **A129,** 157 (1988).

[8] K. E. Kürten and J. W. Clark, Phys. Rev. E, **77,** 046116 (2008).