ECONOPHYSICS Section

NEOCLASSICAL AND NEWTONIAN THEORY OF PRODUCTION: AN EMPIRICAL TEST

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Abstract. *We search for regularities observed in the production of goods. Our first observation is that unit root is found in annual productions in all manufacturing industries in Finland and in Sweden. Thus annual industrial flows of production are observed to follow a first order difference equation (FDE). Industrial flows of production have also exogenous time dependencies, however, and we explain these by changes in the corresponding product prices due to the profit-seeking behavior of firms in the industries. We test the Newtonian theory of production against the neoclassical one in explaining how prices affect industrial flows of production. Our observations are that FDE outperforms the neoclassical theory in explaining the flows of production in every tested industry in both countries, and the Newtonian theory outperforms the FDE in 10 cases of 13 in Finnish industries, and in 14 cases of 18 in Swedish industries. Finally, the Newtonian theory outperforms the neoclassical one in every tested industry.*

Keywords: *Industrial production, Neoclassical economics, Newtonian economics.*

JEL: C51, D21, D24.

1. Introduction

The neoclassical theory of production is presented in all textbooks of economics, even though its accuracy has not much been tested empirically. We have found only one article [1] that studies the empirical performance of the static neoclassical theory of production as it is presented in textbooks of economics. Appelbaum [1] summarizes his results as follows: “We find that except for one case the theory does not pass the proposed tests and furthermore, the primal and dual do not yield the same implications”.

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Even though the neoclassical theory of production as presented in textbooks has not got support in empirical tests, the theory relies on the success obtained in the estimation of neoclassical production functions, see [2, 3]. Even though the theory describes firms' behavior at micro-level, its testing has been made mostly by using industry or macro level data. In [2], the Cobb-Douglas – type of production function is introduced and estimated by aggregate level U.S. data of 1899-1922. The marginal productivity of labor is estimated to be 3/4, and that of capital as 1/4; thus constant returns to scale were observed in production. In spite of the immense problems in constructing a macro-level production function from various heterogeneous production processes [4], this has not stopped the estimation of aggregate level production functions, see e.g. [5, 6].

The results obtained in [2, 3] have been questioned in [7-10], however. These authors show that the observed success in estimating the neo-classical production functions at industrial and aggregate level data has been based on a misunderstanding; the true estimated relation has been an accounting identity, and not a physical production function. Thus all the empirical support for the neoclassical theory has been based on an error. According to Kirman [11], the economic crisis at 2008 showed that the representative agent based equilibrium macro models are useless in forecasting economic behavior and they should be replaced by modeling the interactions between heterogeneous agents. This would change current macroeconomic models – that are based on nineteenth-century physics – to resemble more twentieth-century physics where modeling techniques created for complex systems are applied. To get into that position, however, we need to take similar steps as have been taken in physics, that is, create Newtonian and Lagrangian mechanics for economics and this way enter into statistical and quantum economics. Here we follow this line of research and compare the empirical performance of the Newtonian theory of production introduced in [12] to the neoclassical one by applying Finnish and Swedish data. A similar test has been made in [13], but here our data of Finnish industries is longer and we also have data of the corresponding Swedish industries too.

This study is organized as follows. The data is described in Section 2 and its time series properties are analyzed in Section 3. The two theories to be tested are presented in Section 4, and in Section 5 are the results of this testing. Section 6 is a summary.

2. The data used in the study

We use annual production values at year 2010 prices measuring production volumes in Finnish and Swedish manufacturing industries [14, 15]. The Finnish data is in monetary terms and the Swedish data is indexes; in Finland, a less disaggregated data is available. The data contains the following 18 industries in Finland at 1975-2013, and in Sweden at 1981-2012: C10-C12: Food products, beverages and tobacco products, C13-C15: Textiles, clothing, and leather products, C16: Pulp, wood and wood products except furniture, C17: Paper and paper products, C18: Publishing and printing, C19: Refined petroleum products, coke and nuclear fuel, C20-C21: Chemicals and chemical products, C22: Rubber and plastic products, C23: Other non-metallic mineral products, C24: Basic metals, C25: Fabricated metal products, C26: Computer, electronic and optical products, C27: Electrical equipment, C28: Machinery and equipment, C29: Motor vehicles, trailers, and semi-trailers, C30: Other transport equipment, C31-C32: Furniture and other manufacturing, C33: Repair and installation of machinery and equipment. These sectors cover the whole Finnish and Swedish manufacturing.

We have current value time series $p_{in}q_{in}$and fixed price time series at 2010 prices as $p_{i2010}q_{in}$, $i = 1, \dots, n$, where year 2010 price $p_{i2010}(€/unit)$ is constant, and by $q_{in}(unit/year)$ is denoted the flow of production in industry *i*. The estimates for industrial prices is obtained as $p_{in}q_{in}/(p_{i2010}q_{in}) = p_{in}/p_{i2010}$.

3. Time series properties of the data

The unit root tests in Table 1 show that in production volumes, only in industry C29-C30 in Finland the existence of unit root can be rejected at 5 % critical level. At 1 % critical level, the existence of unit root cannot be rejected in any industrial flow of production in both countries. In prices at 5 % critical level, in Finland only in industry C18 and in Sweden only in industries C13-C15, C27, C28, C29, and C31-C32 the existence of unit root can be rejected. In prices at 1 % level, only in Sweden in industries C13-C15 and C29 the existence of unit root can be rejected.

Table 1 implies that the time series of the annual flows of industrial productions follow the process

$$q_n = a_0 + a_1 q_{n-1} + f(n), \tag{1}$$

where by n is denoted discrete time and if $a_1 = 1$, unit root exists in the series. Function $f(n)$ describes changes in the annual flow of production at time unit n independent of its past behavior. Now, according to Table 1, in all industrial flows of production holds approximately $a_1 = 1$, and then the solution of Eq.(1) is

$$q_n = q_0 + a_0 n + \sum_{i=0}^{n-1} f(1+i), q_0 \ \ constant. \tag{2}$$

An essential difference in the behavior of industrial flows of production depends on whether a_0 deviates from zero. If $a_0 = 0$, the linear time trend $a_0 n$ vanishes in Eq. (2) and then q_n fluctuates solely due to function $f(n)$. To test this matter, we estimate the corresponding difference equations, see Table 2.

Table 1.
ADF unit root tests for the time series

Finland			Sweden		
Industry	*ADF (prob.), volume*	*ADF (prob.), price*	*Industry*	*ADF (prob.), volume*	*ADF (prob.), price*
C10-C12	−0.9 (0.76)	−2.0 (0.27)	C10-C12	−1.3(0.63)	−2.9(0.05)
C13-C15	−0.9 (0.79)	−2.8 (0.06)	C13-C15	−1.2(0.66)	−6.0(0.00)
C16-C17	−1.8 (0.38)	−1.9 (0.33)	C16	−1.6(0.46)	−1.6(0.49)
C18	−1.9 (0.34)	−3.6 (0.01)	C17	−1.3(0.63)	−2.3(0.18)
C19-C22	−0.4 (0.89)	−0.3 (0.92)	C18	−0.1(0.94)	−2.8(0.07)
C23	−1.6 (0.45)	−1.4 (0.55)	C19	−0.5(0.87)	−1.0(0.99)
C24	−1.5 (0.53)	−1.5 (0.52)	C20-C21	−0.7(0.84)	−2.4(0.14)
C25	−1.2 (0.67)	−0.7 (0.83)	C22	−1.7(0.42)	−2.0(0.30)
C26-27	−0.8 (0.81)	−1.8 (0.36)	C23	−0.5(0.87)	−2.2(0.20)
C28	−0.9 (0.78)	−0.8(0.82)	C24	−2.1(0.25)	−0.1(0.94)
C29-C30	−3.4 (0.02)	−1.5(0.54)	C25	−2.1(0.26)	−0.7(0.84)
C31-C32	−2.2 (0.19)	−1.5(0.54)	C26	−0.2(0.93)	−0.7(0.82)
C33	−1.7 (0.43)	−0.2(0.93)	C27	−1.0(0.75)	−3.1(0.04)
			C28	−1.1(0.69)	−3.0(0.04)
			C29	−1.4(0.58)	−4.8(0.00)
			C30	−1.7(0.41)	−0.0(0.95)
			C31-C32	−1.2(0.67)	−3.7(0.01)
			C33	−1.5(0.53)	−2.0(0.29)

Table 2.

First order difference equations (FDE) for annual production volumes

Finland					Sweden				
Industry	a_0 (*T-stat.*)	a_1(*T-stat.*)	*Adj.* R^2	*Akaike*	*Industry*	a_0 (*T-stat.*)	a_1(*T-stat.*)	*Adj.* R^2	*Akaike*
C10-C12	306.0 (1.5)	1.0 (42.1)	0.98	13.6	C10-C12	8.9 (1.3)	0.9 (12.7)	0.84	4.0
C13-C15	−14.9(−0.2)	1.0 (32.7)	0.97	12.8	C13-C15	3.2 (0.5)	1.0 (23.8)	0.95	6.9
C16-C17	1548.1 (2.1)	0.9 (20.8)	0.92	17.1	C16	6.9 (1.5)	0.9 (17.3)	0.91	6.0
C18	156.1 (1.4)	0.9 (15.2)	0.86	12.4	C17	6.1 (1.6)	0.9 (20.6)	0.93	5.3
C19-C22	563.8 (1.5)	1.0 (35.8)	0.97	16.0	C18	−0.0 (−0.0)	1.0 (11.8)	0.82	6.4
C23	300.5 (1.8)	0.9 (12.6)	0.81	13.6	C19	4.9 (1.0)	1.0 (14.3)	0.87	6.1
C24	589.1 (1.9)	0.9 (20.2)	0.92	15.9	C20-C21	3.1 (1.9)	1.0 (44.6)	0.99	5.0
C25	342.3 (1.7)	0.9 (22.3)	0.93	15.4	C22	11.0 (1.9)	0.9 (13.4)	0.86	6.3
C26-27	628.8 (1.4)	1.0 (25.9)	0.95	18.1	C23	5.4 (0.7)	0.9 (9.4)	0.74	6.7
C28	649.9 (1.3)	1.0 (19.1)	0.91	16.9	C24	20.5 (2.2)	0.8 (7.8)	0.66	7.2
C29-C30	931.2 (2.4)	0.7 (6.0)	0.48	14.5	C25	14.4 (2.3)	0.9 (12.1)	0.83	6.9
C31-C32	175.4 (1.4)	0.9 (15.3)	0.86	12.9	C26	2.9 (1.6)	1.0 (29.2)	0.97	6.6
C33	269.4 (1.9)	0.9 (14.2)	0.84	13.4	C27	7.0 (1.1)	0.9 (12.8)	0.85	6.4
					C28	9.7 (1.3)	0.9 (10.4)	0.78	7.6
					C29	9.7 (1.6)	0.9 (12.5)	0.84	8.0
					C30	17.3 (1.7)	0.8 (7.5)	0.65	6.2
					C31-C32	4.8 (1.7)	1.0 (26.3)	0.96	5.8
					C33	5.8 (1.3)	0.9 (17.0)	0.91	6.2

Table 2 indicates that in Finland only industries C16-C17 and C29-C30, and in Sweden only industries C24 and C25 have a linear time trend in annual flows of production. Thus in most cases the time paths in production volumes originate from external sources and they must be explained by economic theories. The FDE explains the industrial flows of production relatively well, and next we test whether economic theories can better these results. Otherwise the conclusion is that annual productions are approximately equal as in previous year, and then economic theories are useless in explaining changes in annual flows of production.

4. The neoclassical and the Newtonian theory of production

In the following we assume that the external source that affects the annual flows of production together with their own history is the price of the produced good. Industrial flows of production are modeled by assuming that an industry operates like a representative firm that produces the whole production in the industry under perfect competition. A more detailed description of industrial productions can be made in the future, but here we do not study differences in competition situations within industries and interdependencies between industries.

The profit function of a representative firm *i* in a perfectly competed industry is assumed as follows

$$\Pi_i(t) = p_i(t)q_i(t) - C_i(q_i(t), t), \quad i = 1, \ldots, n, \tag{3}$$

where $\Pi_i(€/y)$ is the annual profit of the firm, $p_i(€/unit)$ the price of the product of the firm, $q_i(unit/y)$ the annual flow of production of the firm, and $C_i(€/y)$ the cost function of the firm; by y is denoted year. The following cost function is assumed for firm *i*

$$C_i(q_i(t), t) = c_{i0} + c_{i1}q_i(t) + \frac{1}{2}c_{i2}q_i^2(t) - c_{i3}tq_i(t), \tag{4}$$

where $c_{ij}, j = 0, \ldots, 3$ are dimensional parameters; the last term describes decreasing costs with time due to technical development. In the neoclassical theory, the time passage is omitted (i.e. $t = 0$ is assumed in Eq. (4)) and the testable function for the neoclassical theory is

$$\frac{\partial \Pi_i}{\partial q_i} = 0 \quad \Leftrightarrow \quad p_i(t) = C_i'\big(q_i(t)\big) \quad \Leftrightarrow \quad q_i(t) = -\frac{c_{i1}}{c_{i2}} + \frac{1}{c_{i2}}p_i(t). \tag{5}$$

In Newtonian theory, the time passage is explicitly modeled and the following model is estimated for industrial prices

$$p_i(t) = a_{i0} + a_{i1}t + a_{i2}\sin(b_{i0}t + b_{i1}) + a_{i3}\sin(b_{i2}t + b_{i3}), \tag{6}$$

where $a_{ij}, b_{ik}, \ j, k = 0, \ldots, 3$ are dimensional parameters to be estimated. The first sine-function describes normal business cycles with frequency and phase parameters b_{i0}, b_{i1}, respectively, and the second sine-function represents possible longer term cycles. A linear time trend exists in $p_i(t)$ if $a_{i1} \neq 0$.

Substituting Eqs. (4) and (6) in Eq. (3), we get the marginal profit function of firm *i* as

$$\frac{\partial \Pi_i}{\partial q_i} = (a_{i0} - c_{i1}) - c_{i2}q_i(t) + (a_{i1} + c_{i3})t + \\ + a_{i2}\sin(b_{i0} + b_{i1}t) + a_{i3}\sin(b_{i2} + b_{i3}t). \tag{7}$$

The Newtonian equation of production of a profit-seeking firm is (see [13])

$$m_i q_i'(t) = \frac{\partial \Pi_i}{\partial q_i} \ \Leftrightarrow \ m_i q_i'(t) = z_{i0} - c_{i2}q_i(t) + \\ + z_{i1}t + a_{i2}\sin(b_{i0} + b_{i1}t) + a_{i3}\sin(b_{i2} + b_{i3}t), \tag{8}$$

where $z_{i0} = a_{i0} - c_{i1}$, $z_{i1} = a_{i1} + c_{i3}$ and m_i with unit $€ \times y^2/unit^2$ represents the inertia ("mass") of production. Now, $\partial\Pi_i/\partial q_i$ with unit $€/unit$ is the reason ("force") that causes the acceleration in production, $q_i^{''}(t)(unit/y^2)$, of the profit-seeking firm. The solution of Eq. (8) is (see [13])

$$q_i(t) = B_{i0} + B_{i1}t + B_{i2}\sin(b_{i2}t) + B_{i3}\cos(b_{i2}t) + B_{i4}\sin(b_{i0}t + b_{i1}) + B_{i5}\cos(b_{i0}t + b_{i1}) + B_{i6}e^{-\frac{c_{i2}}{m_i}t}, \qquad (9)$$

where $B_{ij}, b_{ik}, j = 0, \ldots, 6,\ k = 0, \ldots, 2$ are dimensional parameters to be estimated.

5. Empirical results

We compare empirically the neo-classical theory in Eq. (5) with the Newtonian one in Eq. (9), and these both are compared to the FDE in Eq.(1). Only statistically significant parameter estimates are reported in Tables 3-8, and the absolute values of pairwise correlations between explaining variables in any model are not allowed to exceed 0.5. The estimated models for industrial prices in Tables 3, 4, show that Eq. (6) works quite well for all industries in both countries. A linear or a more complicated time trend exists in every price, and Eq. (6) explains over 91% of price variation in Swedish industries and over 84% in Finnish industries. The Breusch-Godfrey serial correlation LM test statistic shows positive autocorrelation in residuals of all Finnish industries and in most Swedish industries too. This problem is not analyzed here further because the focus in this paper is in explaining industrial productions and not prices.

Table 3.

The estimated models for Finnish industrial prices.

Industry	*Constant* (*T-test*)	*Time* (*T-test*)	$\sin(b_{i0}t + b_{i1})$ (*T-test*)	$\sin(b_{i2}t + b_{i3})$ (*T-test*)	*Adj.* R^2	*B-G, F- prob.*
C10-C12	−13.3 (−11.8)	0.0 (12.4)	0.2 (12.1)	0.1 (15.5)	0.95	0.00
C13-C15	−0.7(−27.8)		0.0 (3.9)	−1.7 (−60.0)	0.99	0.00
C16-C17	−29.3 (−17.4)	0.0 (17.8)	0.1 (3.9)	−0.0 (−2.8)	0.91	0.00
C18	−33.4 (−38.3)	0.0 (38.1)	−0.4 (−12.4)		0.99	0.00
C19-C22	0.5 (46.3)		−0.1 (−9.4)	−6.6(−24.2)	0.94	0.01
C23	5.3 (46. 2)		8.2 (40.1)		0.98	0.00
C24	−30.8(−13.2)	0.0 (13.5)	0.1 (2.9)		0.84	0.00
C25	−40.7 (−53.9)	0.0(54.8)			0.99	0.00
C26-C27	1.3(80.8)		0.4 (17. 9)		0.89	0.00
C28	−46.0 (−54.2)	0.0 (54.9)			0.99	0.00
C29-C30	21.8 (37. 5)		−0.0 (−2.1)	23.0 (36.4)	0.97	0.00
C31-C32	−42.0 (−84.8)	0.0 (86.2)			0.99	0.00
C33	−47.0 (−84.4)	0.0(85.6)			0.99	0.00

Table 4.

The estimated models for Swedish industrial prices.

Industry	*Constant* (*T-test*)	*Time* (*T-test*)	$\sin(b_{i0}t + b_{i1})$ (*T-test*)	$\sin(b_{i2}t + b_{i3})$ (*T-test*)	*Adj.* R^2	*B-G, F-prob.*
C10-C12	1.4 (27.8)		1.2 (17.3)		0.91	0.00
C13-C15	−102.7 (−85.5)		103.9 (86.4)		0.99	0.10
C16	−113.9 (−23.5)	0.06 (24.1)	0.2 (6.0)		0.95	0.07
C17	−120.4 (−17.3)	0.1 (17.7)	−0.1 (−3.2)	0.4 (5.4)	0.91	0.17
C18	−499.5 (−25.6)		503.7 (25.6)		0.95	0.00
C19	5.4 (38. 2)		−0.3 (−3.0)	4.0 (21.1)	0.94	0.23
C20-C21	−238.1 (−29.2)	0.1 (29.9)	0.3 (5.4)		0.97	0.00
C22	−179.5 (−39.6)	0.1 (40.3)	0.1 (4.3)		0.98	0.00
C23	−175.0 (−84.9)	0.1 (86.2)	−0.2 (−8.2)		0.99	0.00
C24	−169.6 (−13.4)	0.1 (15.0)	0.2 (2.7)	10.2 (5.9)	0.92	0.00
C25	−292.7 (−68.2)	0.1 (69.3)			0.99	0.00
C26	−77.4 (−36.6)		−104.4 (−46.0)	−3.4 (−15.7)	0.99	0.02
C27	−139.7 (−46.2)	0.1 (47.1)	−0.3 (−8.7)		0.99	0.00
C28	−164.0 (−51.9)	0.1 (52.9)	0.2 (8.2)		0.99	0.00
C29	−232.5 (−35.6)	0.1 (36.4)	−0.7 (−15.3)		0.98	0.00
C30	−180.1 (−58.3)	0.1 (59.1)	−0.1 (−3.0)		0.99	0.00
C31-C32	−243.7 (−66.2)	0.1 (67.5)	−0.6 (−15.7)		0.99	0.00
C33	−193.6 (−71.6)	0.1 (72.8)	0.2 (9.2)		0.99	0.02

The estimated frequency and phase parameters of the sine-functions of all models are in the Appendix. The neoclassical models for industrial flows of production in Finland and in Sweden are in Tables 5 and 7 and the corresponding Newtonian models in Tables 6 and 8. According to adjusted R^2 and Akaike info criterion, by pairwise comparison the Newtonian models are all better than the corresponding neoclassical ones for industrial flows of production. In Finland, price has a statistically significant effect in all other neoclassical models except in industry C29-C30, but in industries C13-C15 and C26-C27, the parameter estimate is significantly negative. Thus, in these industries the theory does not work properly. The neoclassical models are, however, not accurate especially in industries C26-C27 and C29-C30 where the models explain only roughly 5% of observed variation. The Newtonian models work quite well in all other Finnish industries except in C18 and in C29-C30 where roughly 60% of observed variation is explained.

Table 5.

The neoclassical models for Finnish industries

Industry	*Constant (T-test)*	p_{it}(*T-test*)	$Adj. R^2$	*Akaike criterion*	*B-G, F-prob.*
C10-C12	3389.2 (3.5)	5659.6 (5.3)	0.42	17.0	0.00
C13-C15	4603.8 (22.4)	−3208.6 (−12.5)	0.80	14.6	0.00
C16-C17	−1304.9(−1.0)	21778.4 (13.5)	0.83	18.0	0.00
C18	1308.5 (8.8)	668.1 (3.4)	0.22	14.2	0.00
C19-C22	3529.2 (3.6)	15328.9 (10.7)	0.75	18.3	0.00
C23	1078.0 (7.5)	1835.3 (9.4)	0.70	14.1	0.00
C24	−293.4 (−0.4)	9381.7 (8.3)	0.64	17.4	0.00
C25	−1277.0 (−4.6)	7930.4 (21.2)	0.92	15.5	0.00
C26-C27	19883.0 (3.2)	−8320.4 (−1.9)	0.06	21.0	0.00
C28	874.6 (1.4)	12691.8 (14.2)	0.84	17.5	0.00
C29-C30	2910.9 (13.6)	462.7 (1.6)	0.04	15.1	0.00
C31-C32	1453.8 (8.4)	873.9 (3.7)	0.25	14.7	0.00
C33	1128.5 (12.8)	1691.3 (13.6)	0.83	13.6	0.00

Table 6.

The Newtonian models for Finnish industries.

Industry	*Constant* (*T-test*)	*Time* (*T-test*)	$\sin(b_{i2}t)$ (*T-test*) $\cos(b_{i2}t)$ (*T-test*)	$\sin(b_{i0}t + b_{i1})$ (*T-test*) $\cos(b_{i0}t + b_{i1})$ (*T-test*)	*Adj.*R^2	*Akaike criterion*	*B-G, F, prob.*
C10-C12	−249152.2 (−50.2)	129.2 (51.9)		−172.6 (−4.3)	0.99	13.2	0.01
C13-C15	131805.1 (24.2)	−65.1 (−23.8)		−300.8 (−6.6) −217.3 (−5.0)	0.94	13.4	0.00
C16-C17	−671619.0 (−21.4)	344.7 (21.8)		1921.0(7.1) −545.8 (−2.2)	0.95	16.7	0.03
C18	−13795.5 (−2.3)	7.8 (2.7)		−299.9 (−6.6)	0.57	13.6	0.00
C19-C22	11107.6 (92.8)		 −2199.9(−14.3)	6897.3 (40.4)	0.98	15.9	0.19
C23	−61693.4 (−9.7)	32.1 (10.1)		−225.1 (−4.8) −207.9 (−4.1)	0.83	13.6	0.00
C24	−411073.4 (−23.3)	209.3 (23.7)	 −994.8 (−6.6)	− 867.9 (−6.0)	0.94	15.7	0.72
C25	−332281.3 (−23.1)	168.8 (23.4)	305.5 (2.7)	 −468.3 (−3.9)	0.94	15.4	0.00
C26-C27	−1404851.0 (−19.5)	708.6 (19.6)	−3090.9 (−5.4)	−2263.4 (−3.8)	0.92	18.6	0.00
C28	−588035.7 (−19.3)	299.4 (19.6)	−954.8 (−3.7)	668.9 (2.8)	0.92	16.9	0.00
C29-C30	−19212.0 (−2.4)	11.3 (2.8)	−410.8 (−6.6)	−254.9 (−4.1) −194.7 (−3.0)	0.64	14.2	0.01
C31-C32	2015.4 (84.9)		318.8 (8.4) −282.4 (−9.7)	92.2 (2.8) −260.3 (−8.1)	0.89	12.8	0.00
C33	−78179.0 (−14.9)	40.3 (15.3)	 −100.3 (−2.4)	 −137.4 (−3.3)	0.87	13.4	0.87

Table 7.

The neoclassical models for Swedish industries.

Industry	*Constant* (*T-test*)	p_{it} (*T-test*)	*Adj.* R^2	*Akaike criterion*	*B-G, F-prob.*
C10-C12	79.3 (30.5)	7.1 (6.2)	0.55	5.1	0.00
C13-C15	286.4 (33.0)	−144.5 (−16.9)	0.90	7.7	0.00
C16	25.7 (3.0)	21.4 (7.2)	0.62	7.5	0.00
C17	24.7 (4.0)	20.0 (9.7)	0.75	6.7	0.00
C18	146.7 (23.8)	−13.8 (−5.0)	0.43	7.5	0.00
C19	53.1 (15.2)	8.2 (7.1)	0.62	7.3	0.00
C20-C21	−31.0 (−3.9)	20.3 (13.1)	0.85	7.4	0.00
C22	43.1 (7.1)	13.6 (7.8)	0.66	7.2	0.00
C23	64.4 (8.6)	6.1 (2.4)	0.13	7.8	0.00
C24	64.9 (10.4)	9.6 (4.7)	0.41	7.8	0.00

C25	32.7 (5.7)	11.8 (9.9)	0.76	7.4	0.00
C26	130.5 (25.6)	−4.5 (−18.6)	0.92	7.5	0.00
C27	48.0 (5.9)	15.0 (5.1)	0.44	7.7	0.00
C28	11.3 (1.2)	22.2 (7.5)	0.64	8.0	0.00
C29	−25.5 (−1.4)	19.3 (5.9)	0.52	9.1	0.00
C30	76.7 (20.0)	6.3 (4.4)	0.37	6.8	0.00
C31-C32	−8.2 (−1.2)	18.4 (12.2)	0.83	7.3	0.00
C33	27.7 (4.4)	15.5 (8.7)	0.71	7.3	0.00

Table 8.

The Newtonian models for Swedish industries.

Industry	*Constant* (*T-test*)	*Time* (*T-test*)	$\sin(b_{i2}t)$ (*T-test*) $\cos(b_{i2}t)$ (*T-test*)	$\sin(b_{i0}t + b_{i1})$ (*T-test*) $\cos(b_{i0}t + b_{i1})$ (*T-test*)	*Adj.* R^2	*Akaike criterion*	*B-G, F-prob.*
C10-C12	−737.7 (−10.8)	0.4 (12.2)	−1.1 (−2.4)	 1.2 (2.6)	0.83	4.12	0.00
C13-C15	7034.3 (29.8)	−3.5 (−29.2)	−12.7 (−8.2)	−4.2 (−2.7) 3.6 (2.3)	0.97	6.6	0.11
C16	−3206.1 (−21.6)	1.6 (22.1)	−2.3 (−2.5) −9.5 (−9.8)	−3.7 (−3.9) 1.9 (2.2)	0.95	5.6	0.02
C17	−2848.9 (−27.5)	1.5 (28.3)	 −2.3 (−2.5)	−5.5 (−6.0)	0.97	4.7	0.59
C18	1551.3 (8.0)	−0.7 (−7.4)	−9.5 (−7.7)	3.8 (3.6) 4.6 (4.6)	0.91	5.8	0.11
C19	− 2781.0 (−22.3)	1.4 (22.9)		1.7 (2.2) 3.2 (3.9)	0.95	5.3	0.24
C20-C21	65.8 (36.2)		−7.6(−3.5) −41.4 (−66.0)	 6.0 (11.2)	0.99	4.3	0.01
C22	− 3082.7 (−15.3)	1.6 (15.7)	8.9 (6.6)	−4.6 (−3.6)	0.89	6.2	0.15
C23	−1217.3 (−5.9)	0.6 (6.3)	 13.3 (10.2)	−3.8 (−2.9) −3.1 (−2.3)	0.83	6.3	0.00
C24	64.6 (22.7)		 7.5 (4.2)	 39.5 (10.7)	0.80	6.8	0.29
C25	−3827.9 (−11.9)	2.0 (12.2)		5.3 (2.7) 5.4 (2.6)	0.83	7.0	0.00
C26	−7341.4 (−28.1)	3.7 (28.3)	4.6 (2.8) 4.8 (3.0)	−5.0 (−2.9) 8.1 (4.7)	0.96	6.8	0.00
C27	−2708.9 (−12.8)	1.4 (13.2)		10.3 (7.2) − 5.2(−3.9)	0.87	6.3	0.01
C28	−4231.0 (−12.7)	2.2 (12.9)		−9.6 (−4.2)	0.85	7.2	0.01
C29	−6664.7 (−15.0)	3.4 (15.2)		−9.4 (−3.2) 14.7 (5.3)	0.89	7.7	0.04
C30	92.5 (137.5)		−7.7(−8.0) 7.4 (7.8)	3.3 (3.4)	0.81	5.6	0.14
C31-C32	66.2 (12.4)		3.7 (4.0) −5.8 (−5.8)	−12.8 (−2.1) −44.8 (−29.2)	0.97	5.6	0.04
C33	−3043.9 (−13.7)	1.6 (14.0)		5.5 (3.6) −6.3 (−4.3)	0.88	6.5	0.00

In Finland, we obtained a statistically significant parameter for price in all other neoclassical models except in industry C29-C30, but in industries C13-C15 and C26-C27, the parameter estimate is significantly negative. Thus, in these industries the theory does not work properly. The neoclassical models are, however, not accurate especially in industries C26-C27 and C29-C30 where the models explain only roughly 5 % of observed variation. The Newtonian models are quite accurate in all other Finnish industries except in C18 and in C29-C30, where only roughly 60 % of observed variation is explained by the models.

In the case of Sweden, in all estimated neoclassical models a statistically significant estimate for price is obtained, but in industries C13-C15, C18, and C26, the estimate is significantly negative. On the average the neoclassical theory works better in Sweden than in Finland, and only in industry C23 in Sweden the model works badly, i.e. adjusted $R^2 = 0.13$. In terms of adjusted R^2, in Sweden, however, in the best neoclassical models a negative parameter estimate is obtained for price. This further questions the empirical performance of the neoclassical theory. The Newtonian models are quite good for all Swedish industries, and in no industry less than 81 % of observed variation in production flows is explained by the models. One further advantage of the estimated Newtonian models as compared with the neo-classical ones is that the former can be used in forecasting simply by increasing time in the models. Using the neoclassical models in forecasting, on the other hand, industrial prices must be forecasted as well.

According to the Breusch-Godfrey serial correlation LM test statistic, positive autocorrelation problem exists in the residuals of all estimated functions. This shows that the estimated functional forms do not follow the observed data accurately enough. The autocorrelation problem is worse in the neoclassical than in the Newtonian models, however. This implies that there is room to better the obtained results. Two best neoclassical models for both countries are graphed in Figures 1, 3, 5, and 7, and the corresponding Newtonian ones in Figures 2, 4, 6, and 8.

6. Conclusions

We tested a Newtonian type of model for industrial flows of production against the neoclassical one, and these both were compared to the first order difference equation (FDE). Our observation is that the Newtonian theory and FDE outperform the neoclassical one in every industry in both countries, Finland and Sweden. In pairwise comparisons, the Newtonian theory is better than FDE in 24 cases of 31. These results imply that the neoclassical theory is not flexible enough to explain the industrial flows of production, and at worst it can explain only 4 % of observed variation of annual production in one industry. The Newtonian theory is more flexible, and at worst it can explain 57 % of observed variation in annual production in one industry.

Most of the estimated models were shown to have positive autocorrelation problem in residuals. This implies that more accurate functional forms can be found. In the neoclassical theory, however, there are no means to improve it. Game theory, on the other hand, could be useful in creating models where interdependencies between firms' productions are incorporated. In estimating this kind of models, however, firm level production data would be required, which is seldom available. On the other hand, interdependencies between industries may be modeled by taking account possible input-output -relations between industries. It would be interesting if similar tests as we made here are repeated with quarterly or monthly data, or with different countries and different levels of aggregation.

APPENDIX

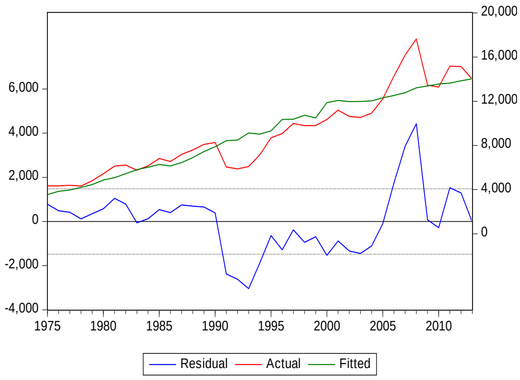


Figure 1. The neoclassical model for C28 in Finland.

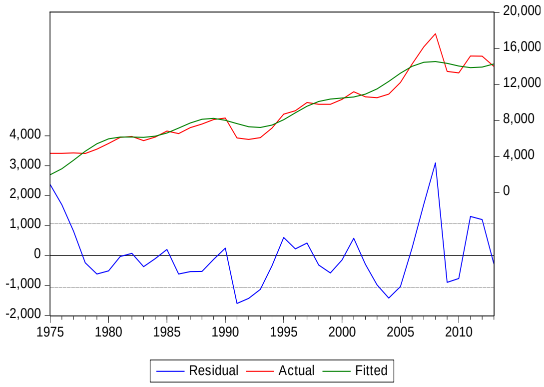
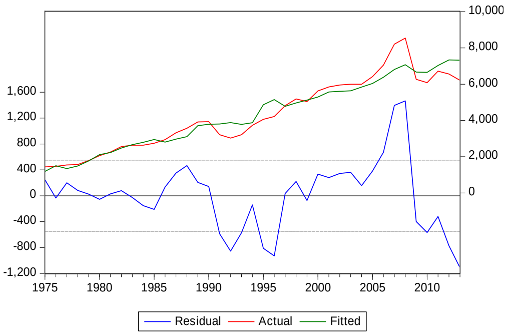


Figure 2. The Newtonian model for C28 in Finland.



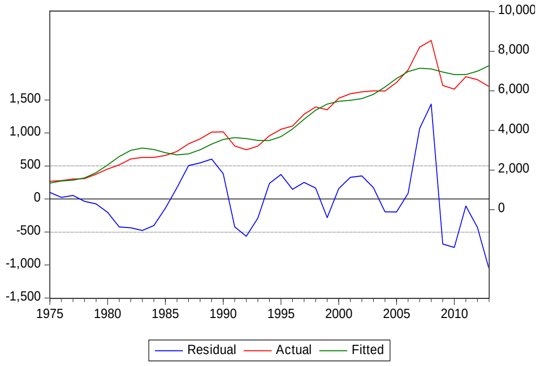
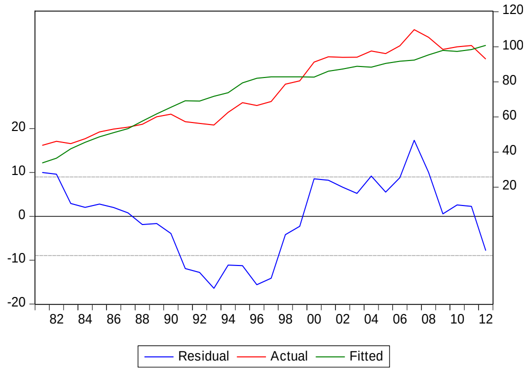


Figure 3. The neoclassical model for C25 in Finland.

Figure 4. The Newtonian model for C25 in Finland.



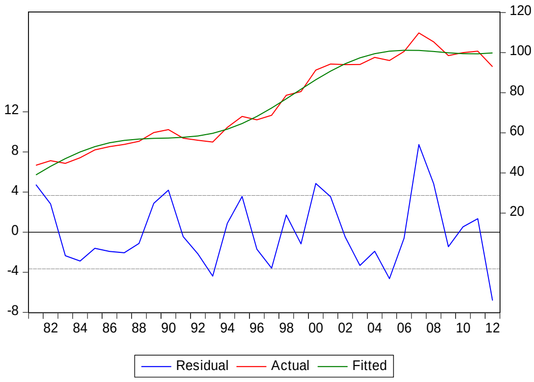
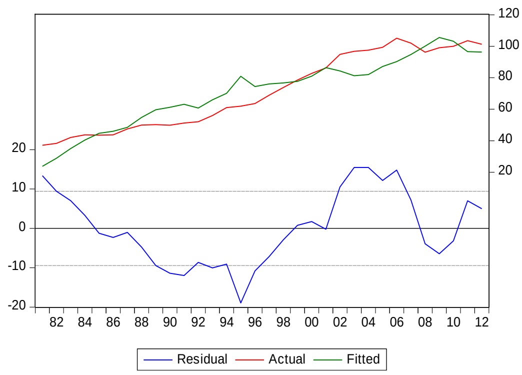


Figure 5. The neoclassical model for C31-C32 in Sweden.

Figure 6. The Newtonian model for C31-C32 in Sweden.



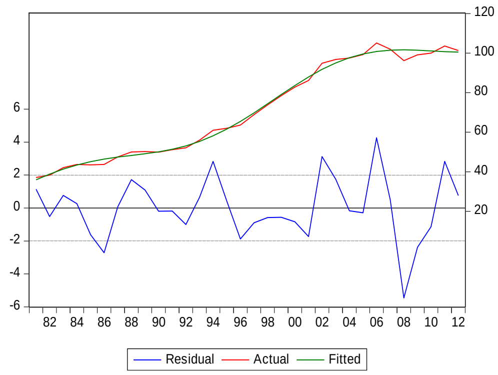


Figure 7. The neoclassical model for C20-C21 in Sweden.

Figure 8. The Newtonian model for C20-C21 in Sweden.

The estimated frequency and phase parameters of the sine-functions in models for industrial prices in Finland are: $b_{C10C120} = -18.77$, $b_{C10C121} = 0$, $b_{C10C122} = 0.2693$, $b_{C10C123} = 0$, $b_{C13C150} = 0.021$, $b_{C13C151} = -0.009$, $b_{C13C152} = -94.7$, $b_{C13C153} = 0.05$, $b_{C16C170} = -728.49$, $b_{C16C171} = 0.36$, $b_{C16C172} = 427.66$, $b_{C16C173} = -0.21$, $b_{C180} = -18.8$, $b_{C181} = 0$, $b_{C19C220} = 4429.44$, $b_{C19C221} = -2.2$, $b_{C19C222} = -0.00318$, $b_{C19C223} = 0.04$, $b_{C230} = -0.00296$, $b_{C231} = -5840$,$b_{C240} = 483.34$, $b_{C241} = 0.01$, $b_{C26C270} = 0.1504$, $b_{C26C271} = 141.6$, $b_{C29C300} = 24.478$, $b_{C29C301} = 0$, $b_{C29C302} = 0.00235$, $b_{C29C303} = 13$.

The frequency and phase parameters of the sine-functions in models for industrial prices in Sweden are: $b_{C10C120} = 0.06$, $b_{C10C121} = 0.43$, $b_{C13C150} = 0.0039$, $b_{C13C151} = 0.01$, $b_{C160} = -107.11$, $b_{C161} = 0.05$, $b_{C170} = -130.88$, $b_{C171} = 0.07$,$b_{C172} = -106.7$, $b_{C173} = 0.05$,$b_{C180} = -0.0015$, $b_{C181} = -498$, $b_{C190} = 5.095$, $b_{C191} = 0$,$b_{C192} = -0.08127$, $b_{C193} = 3.2$, $b_{C20C210} = 0.2$, $b_{C20C211} = 0.01$,$b_{C220} = 0.31$, $b_{C221} = 0.315$, $b_{C230} = 0.125$, $b_{C231} = 0.125$,$b_{C240} = 332.59$, $b_{C241} = -0.16$, $b_{C242} = 0.029$, $b_{C243} = -2.9$, $b_{C260} = 1445.11$, $b_{C261} = -0.71$, $b_{C262} = 0.2988$, $b_{C263} = -1564$, $b_{C270} = -138.3568$, $b_{C271} = 0.07$, $b_{C280} = 0.268$, $b_{C281} = 13.38$,$b_{C290} = 0.188$, $b_{C291} = 0.06$, $b_{C300} = -200.71$, $b_{C301} = 0.1$, $b_{C31C320} = 0.1121$, $b_{C31C321} = -5.2$, $b_{C330} = -590.37$, $b_{C331} = 0.29$.

The estimated frequency and phase parameters of the sine-functions in Newtonian models for production flows in Finland are: $b_{C10C120} = -248675.6$, $b_{C10C121} = 128.7$, $b_{C13C150} = 129125.5$, $b_{C13C151} = -64$, $b_{C16C170} = -648946.5$,$b_{C16C171} = 333$, $b_{C16C172} = 798253.4$, $b_{C180} = -6911.7$,$b_{C181} = 0.15$, $b_{C19C220} = -733605.8$, $b_{C19C221} = 37$, $b_{C19C222} = -56.7$,$b_{C230} = -69008.6$, $b_{C231} = 35.2$, $b_{C240} = -380314.8$, $b_{C241} = 19$, $b_{C242} = 245.3$, $b_{C250} = -325117.4$, $b_{C251} = 16$, $b_{C252} = -329923$, $b_{C26C270} = -1279733.8$, $b_{C26C271} = 64$, $b_{C26C272} = 396.0$, $b_{C280} = -598731.7$, $b_{C281} = 303.4$, $b_{C282} = -601319.4$, $b_{C29C300} = -13491.0$, $b_{C29C301} = 8.2$, $b_{C29C302} = 43.6$, $b_{C31C320} = -32760.9$, $b_{C31C321} = 17.4$, $b_{C31C322} = -37.6$, $b_{C330} = -74632.6$, $b_{C331} = 3$, $b_{C332} = -79676.6$.

The estimated frequency and phase parameters of the sine-functions in Newtonian models for production flows in Sweden are: $b_{C10C120} = -734.5$, $b_{C10C121} = 0.01$,$b_{C10C122} = 1.1$, $b_{C13C150} = 7358.1$, $b_{C13C151} = -3.6$, $b_{C13C152} = -0.3$, $b_{C160} = -3104.7$, $b_{C161} = 1.6$, $b_{C162} = -0.3$, $b_{C170} = -2707.7$, $b_{C171} = 1.4$, $b_{C172} = -0.3$, $b_{C180} = 1577.7$,$b_{C181} = -0.7$,$b_{C182} = -0.2$,$b_{C190} = 1551.4$, $b_{C191} = -0.7$, $b_{C20C210} = 10982.7$, $b_{C20C211} = -5$, $b_{C20C212} = -0.1$, $b_{C220} = -2558$, $b_{C221} = 1$,$b_{C222} = -0.4$, $b_{C230} = -1130.4$, $b_{C231} = 0.6$, $b_{C232} = 0.3$,$b_{C240} = -2626.3$, $b_{C241} = 1.3$, $b_{C242} = -0.3$, $b_{C250} = -3442.8$, $b_{C251} = 1.7$, $b_{C260} = -7018$, $b_{C261} = 3.5$, $b_{C262} = 0.7$, $b_{C270} = 42662.5$, $b_{C271} = -25$, $b_{C280} = -4009$, $b_{C281} = 2$, $b_{C290} = -5843.7$, $b_{C291} = 3.5$, $b_{C300} = 13068.4$, $b_{C301} = -5$, $b_{C302} = 0.2$, $b_{C31C320} = -4467.4$, $b_{C31C321} = 2$, $b_{C31C322} = 0.3$, $b_{C330} = -3147.6$, $b_{C331} = 2.5$.

REFERENCES

[1] E. Appelbaum, *Testing neoclassical production theory*, Journal of Econometrics Vol. **7,** Issue 1, 87-102 (1978).

[2] C. W. Cobb, P. H. Douglas, *A Theory of Production*, American Economic Review 18 (Supplement), 139-165 (1928).

[3] P. H. Douglas, *Are There Laws of Production*?, American Economic Review 38, 1-41 (1948).

[4] F. M. Fisher, *The Existence of Aggregate Production Functions*, Econometrica Vol. **37,** No. 4 (1969).

[5] N. Epstein & C. Macchiarelli, *Estimating Polan's Potential Output: A Production Function Approach*, IMF Working Papers WP/10/15 (2009).

[6] Md. M. Hossain, A. K. Majumder and T. Basak, *An Application of Non-Linear Cobb-Douglas Production Function to Selected Manufacturing Industries in Bangladesh,* Open Journal of Statistics 2, 460-468 (2012).

[7] A. Shaikh, *Laws of Production and Laws of Algebra: The Humbug Production Function*, The Review of Economics and Statistics 56,115-120 (1974).

[8] J. Felipe, *Does the aggregate production function imply anything about the laws of production*?, Applied Economics Letters 5, 543-547 (1998).

[9] J. Felipe, R. Hasan, J. S. L. McCombie, *Correcting for biases when estimating production functions: an illusion of the laws of algebra*?, Cambridge Journal of Economics 32, 441-459 (2008).

[10] J. Felipe, J. McCombie, *Why the data tell us nothing about the importance of increasing returns to scale and externalities to capital*, Economia e Sosiedade, Campinas 17 (Numero especial), 655-675 (2008).

[11] A. Kirman, *The Economic Crisis is a Crisis for Economic Theory*, CESifo Economic Studies, Vol. **56,** 498-535 (2010).

[12] M. Estola, *A Dynamic theory of a firm: An Application of "Economic Forces*", Advances in Complex Systems Vol. **1,** N:o 1, 163-176 (2001).

[13] M. Estola & A. Dannenberg, *Testing the neo-classical and the Newtonian theory of production,* Physica A 391, 6519-6527 (2012).

[14] National Accounts, Statistics Finland, http://www.stat.fi.

[15] National Accounts, Statistics Sweden, http://www.scb.se/en_